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To Miss Kinnaird

with the Author's most respectful Compliments

1821

A
Companion
TO EVERY TREATISE
ON
ARITHMETIC,

IN THE FORM OF QUESTION AND ANSWER,

IN WHICH

The Science is demonstrated in a clear and familiar manner, with the view of assisting the Teacher, and rapidly advancing the progress of the Pupil:

TO WHICH IS ADDED

An Appendix, containing useful Tables, and numerous Questions for exercise in leisure hours.

BY

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The true and only method of promoting Science, is to communicate it with clearness and precision.

Gregory.

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1821.

PREFACE.

The importance of Figures is so well known, that a work calculated to fix the theory, render the application easy, and insure accuracy and dispatch in the practice, requires no apology to the Public.

Many small tracts in the way of question and answer* have been published for the young arithmetician; but none have ever come under my notice, that could, with any degree of propriety, be put into the hands of a beginner, they being for the most part very obscure.

The pupil is necessitated to take a great deal for granted; and must rigidly follow the rules without (to use a common saying) knowing why or wherefore; which brings to my recollection an anecdote of a celebrated Elocutionist of my acquaintance, whose pupils could only be instructed by imitation. On being asked by a young student how he should be able to

* A useful little tract has been published by that very able Accountant Mr. James Morrison; formerly Master of the Mercantile Academy, Glasgow; now Master of the Mercantile Academy, Leeds; whom I have had the honour of assisting in tuition while in Glasgow.

acquit himself in the various inflections to different readings when alone; acknowledging, that although he found no difficulty whatever in imitating his master; (who always read before him) yet he confessed that he could proceed with very little confidence in himself, without his presence; the only reply the young gentleman received was "*read as I read.*"

I fear that many of our teachers of arithmetic, likewise, follow too much the plan of *calculate as I calculate*, without giving any sufficient reason, as they ought to do, for every rule in calculation; such methods are, by no means, satisfactory to the ingenious and inquiring mind.

Arithmetic, when rationally taught, affords, perhaps, to the youthful mind, the most advantageous exercise of its reasoning powers; and that for which the human intellect becomes most early ripe: while the more advanced parts of the science may try the energies of an understanding the most vigorous and mature.

It is of much less consequence how far we proceed, than that we make ourselves masters of the ground as far as we do proceed: whatever we learn, whether little or much, should be learned thoroughly. "A smattering of information about a variety of subjects" says a profound Arithmetician* " is calculated to ex-

* Mr. Walker, the celebrated Lecturer on the Philosophy of Arithmetic, &c. and formerly Fellow of Dublin College, from whose lectures I have derived much benefit.

cite that vanity and presumption of knowledge which is repressed by a radical acquaintance with the most elementary principles of some one science."

Very few of the numerous pupils of both sexes whom I have had the honour of teaching arithmetic, could produce a proper reason for the most simple operations they had to perform. In order, therefore, to remedy this defect, I have drawn up the following Treatise in the form of question and answer; which method I think by far the best for instilling into youthful minds the principles of any science.

Nothing is to be found in the Companion to every Treatise on Arithmetic, but what is absolutely necessary to be fixed in the mind of the pupil; "this particular" says an eminent Author,* "seems to have been greatly neglected by most of our arithmetical writers; although I am persuaded that a proper attention to it, would be of great service both to tutor & pupil."

As many learn this useful branch of education at an early age; the rules and definitions are therefore simplified, as much as the nature of the subject will admit; and all the terms made use of in the science are completely explained.

I have studiously avoided algebraical demonstration; which, no doubt, is more natural and elegant; but, as the young student is not yet supposed to have any acquaintance with

* Bonnycastle.

this noble science, I have considered it best to make use of materials with which the pupil is already engaged. As Long Division is the greatest difficulty which the young arithmetician has to encounter, particularly if the divisor be a high number, I have inserted such rules as will enable him to divide by the highest numbers as easily as by any of the nine digits.

The order in which the different rules should be taught, is a matter entirely arbitrary; and, therefore, I shall give but few directions on that head. The only thing that I shall venture to say is, that when the pupil has acquired a perfect knowledge of Addition, Subtraction, Multiplication and Division of simple numbers; he should next be directed to Reduction, then to the Compound rules of Addition, Subtraction, Multiplication, and Division; then the Doctrines of Arithmetical and Decimal Fractions; afterwards, Proportion, Practice, Interest, &c. &c.

The rules, however, are so disposed, as to have but little dependence on each other; and, consequently, every teacher is left to his own discrimination in that respect.

In the appendix I have given a list of the tables of money, weights, and measures which are used in the British Empire; and a Multiplication table, so explained as to enable the pupil to get this grand prop of calculation by heart in half the usual time.

I would advise that every child be thoroughly initiated in it, before proceeding to calculation of any kind. It is not long before children can be made acquainted with Addition and Subtraction of simple numbers; and were it not for the very imperfect manner in which they in general get this table, Multiplication and Division, which are but other names for Addition and Subtraction,* would not occupy much longer time. It is, therefore, indispensably necessary to have as early and correct a knowledge of it as possible. I have likewise added a few amusing questions for exercise in leisure hours.

This small Treatise, will, it is presumed, be found a useful, as well as a very necessary companion to every Treatise on the subject of Arithmetic; then may I be allowed to say to every Author, in the language of the Poet:—

"Oh while along the stream of time thy name
"Expanded flies, and gathers all its fame;
"Say, shall my little bark attendant sail?
"Pursue the triumph, and partake the gale?"

Pope.

How far I have succeeded, or wherein I have failed, in my present attempt to render Arithmetic a pleasant study, must be referred to the determination of the competent and candid reader.

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* See Multiplication and Division.

A
COMPANION
TO
EVERY TREATISE
ON
Arithmetic.

I. INTRODUCTION.

1. WHAT is Arithmetic?

It is a science which teaches the art of accounting, and all the powers and properties of numbers.

2. What is Arithmetic considered both as a science and as an art?

Arithmetic, as a science, explains the properties of numbers; and, as an art, teaches the method of computing by them.

3. How does it appear that Arithmetic is both a science and an art?

Arithmetic is a science, because its rules are capable of being demonstrated; and it is an art, because it applies those rules to useful calculations, in the concerns of life.

4. What are the fundamental rules of Arithmetic?

The fundamental rules of Arithmetic are Notation, Numeration, Addition, Subtraction, Multiplication, and Division.

5. To how few may all the operations of Arithmetic be reduced?

To two, Addition and Subtraction; for Multiplication and Division are but abridged methods of performing Addition and Subtraction. [See VI. VII.]

6. By what symbols are numbers represented?

By the ten following characters: viz.—
1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

7. What are these characters called?

The nine first are called significant figures, or digits; and increase in a progression of ones; but the last, or cipher, has no value of its own.

II. NOTATION AND NUMERATION.

8. What is Notation?

It is the method of *writing* down a number in figures, as 126.

9. What is Numeration?

It is the art of *reading* a number expressed in figures; as one hundred and twenty six.

10. What is the general proportion of simple numbers?

Their proportion is tenfold; for they *increase* in that proportion from right to left; and *decrease* in the same proportion from left to right. *Ten*, therefore, in any position, are but equivalent to *one* in the next position towards the left; and *one* is equal to *ten* in the next position towards the right.

11. What is the probable reason for *ten* being used as the scale for simple numbers?

It is obviously to be accounted for from the natural circumstance of our fingers, they being, in the origin of society, the readiest instrument to assist Numeration.

12. Since the cipher has no value of its own, what then is its use?

Though it has no value of its own, it is used for setting the other nine, higher or lower in the scale of numbers; and that in a tenfold proportion.

13. How do you read or express the value of any given number?

By dividing the number into periods of six figures each, from the units place; and these again into half periods; reckoning the first half period so many units; the second so many thousands; the third so many millions, &c.

14. How do you express in figures any number proposed in words?

By beginning at the left hand and writing towards the right; setting every figure in the period and place which the verbal expression points out; and supplying the vacant places with ciphers.

15. How many periods are there altogether in figures?

No definite number; but seldom more than the following:—Units, Millions, Billions, Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, and Decillions.

16. To whom are we indebted for our present mode of numeral Notation?

To the Arabs; who do not pretend to be the inventors of the characters: but acknowledge that they received them from the Indians.

17. How was it introduced into England?

It was brought by the Moors into Spain; and John of Basingstoke, Archdeacon of Leicester, is supposed to have introduced it into England, about the middle of the eleventh century.

III. EXPLANATION OF THE CHARACTERS MADE USE OF IN ARITHMETIC.

= The sign of equality; signifies that the amount, product, remainder, or in one word, the results, however taken, on each side of it, are equal; as 20 shillings = 1 pound, &c.

+ Means plus, or more; and signifies that the numbers between which it is placed are to be added together; as $2 + 2 = 4$.

— Means minus, or less; and signifies that the number following it is to be subtracted from that which precedes it; as $3 - 2 = 1$.

× Signifies that the numbers between which it stands are to be multiplied together; as $12 \times 6 = 72$.

÷ Signifies that the number which precedes it is to be divided by that which follows it; as $12 \div 6 = 2$.

: :: : Is the sign of proportion; $2 : 4 :: 3 : 6$, signifies that 2 bears the same proportion to 4, that 3 bears to 6.

√ Signifies the square root of whatever number it precedes; as $\sqrt{9} = 3$; and $\sqrt[3]{}$ signifies the cube root; as $\sqrt[3]{8} = 2$, &c.

3^2 signifies the square of 3 which is 9; and the 3^3 is the cube of 3 which is 27, &c.

$\overline{6 + 4} \times 2 = 20$; means 6 added to 4, and this result multiplied by $2 = 20$.

∴ Signifies *therefore* as $2 + 2 = 4$, ∴ $4 = 2 + 2$.

IV. ADDITION.

18. What is Addition?

It is that operation by which we find the sum of two or more given numbers?

19. How do you set down integers to be added?

By placing the numbers one below another in such order, that all the units, all the tens, all the hundreds, &c. may stand in their respective columns.

20. What is an integer?

It is a whole number, or a collection of ones; as 1, 3, 176, &c.

21. How do you add numbers?

By ascertaining the sum of each column separately, and setting down the right-hand figure, carrying the others to the next superior place; which is the same as carrying one for every ten; under the last, or highest place, the full sum must be put down.

22. Why do you carry by ten in adding integers?

Because ten, in any position, are but equivalent to one, in the next position towards the left hand.

23. How do you write down compound numbers to be added?

By placing numbers of the same name under each other; as pence under pence; shillings under shillings; and pounds under pounds, &c.

24. What is a compound number?

A compound number is composed of two, or more denominations; as pounds and shillings; or cwts. qrs. lbs. &c.

25. Do you carry by ten in compound numbers, as you do in simple ones?

No; I sometimes carry by one number; and sometimes by another; for, in money, I carry by 4 at farthings; by 12 at pence; and by 20 at shillings, &c.

26. How do you prove Addition?

By repeating the operation, beginning at the top, and adding downwards.

V. SUBTRACTION.

27. What is Subtraction?

It is that operation by which we find the difference between two given numbers.

28. What are those numbers called?

The greater number is called the *minuend;* and the less the *subtrahend.*

29. How do you set down integers to be subtracted?

I place the subtrahend below the minuend, in the same order as directed in Addition.

30. How do you proceed when any figure in the lower line is greater than its corresponding figure in the upper line?

I add ten to the upper figure, subtract the lower from the amount; and carry one to the next figure in the lower line.

31. Since you add *ten* to the upper figure, why do you only carry *one* to the next figure in the lower line?

Because, from the nature of numbers, *ten* in any column, make but *one* in the next column towards the left hand. [See 10.]

32. How do you set down compound numbers to be subtracted?

By placing like denominations under like; as in Addition,

32. How do you prove Subtraction?

By adding the difference to the less number it produces the greater, or by subtracting it from the greater, the result, will be the less.

VI. MULTIPLICATION.

34. What is Multiplication?

It is that operation by which we find the amount of any two numbers; when the one is to be reckoned as many times as there are units in the other; and it serves instead of many additions. Multiplication is, therefore, a compendious method of performing Addition.

35. What are the numbers employed in Multiplication called?

The number to be multiplied is called the *multiplicand;* that by which we multiply the *multiplier;* and the result is called the *product.*

36. How do you proceed in Multiplication when the multiplier is not more than 12?

By repeating the units place of the multiplicand as many times as there are units in the multiplier; carrying as in Addition; and proceeding in the same manner with the other places of the multiplicand.

37. How do you multiply when the multiplier is an incomposite number beyond 12?

I multiply by each figure separately, observing to place the first figure of each product directly under the figure I am multiplying by; the amount of the several products is the answer.

38. What is a prime or incomposite number?

A prime or incomposite number is that which is not divisible exactly, but by itself & unity.*

39. How do you multiply when the multiplier is a composite number beyond 12?

I may either multiply in the manner described in No. 37; or by the component parts of the number.

40. What is a composite number?

A composite number is such a one as can be disposed into two or more component parts; thus, 3 and 8, or 4 and 6, or 2 and 12 are the component parts of the composite number 24.

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* Unity signifies *one.*

How do you multiply by 10, 100, 1000, &c.?

To multiply by 10 raises the multiplicand one step higher in the scale of numbers; the multiplicand, therefore, with a cipher annexed, is the product; to multiply by 100 raises the multiplicand two steps higher; therefore *two* ciphers annexed produce the product; and to multiply by 1000 is the multiplicand with three ciphers annexed, &c.

42. How do you multiply by a mixed number?‡

I multiply by the integer as before directed, and take a proportional part of the multiplicand for the fraction. [See Divison.]

43. How do you multiply compound numbers?

By multiplying the several denominations of the multiplicand, beginning at the lowest, and carrying as in compound Addition.

44. How do you prove Multiplication?

In simple numbers I make the former multiplier the multiplicand, and the multiplicand the multiplier; which will produce the same product as before,—and

In compound numbers, I divide the product by the multiplier; which will produce the multiplicand.

‡ The pupil may omit this query till he has acquired a knowledge of Division.

VII. DIVISION.

45. What is Division?

It is that operation by which we find how often one number is contained in another; and it is a short method of performing Subtraction; for as many times as one number is contained in another, is the number of times that that number can be subtracted from the other.

46. What names are given to the numbers made use of in Division?

The number to be divided is called the *dividend;* that by which we divide the *divisor;* and the number of times the divisor is contained in the dvidend, the *quotient;* what is over is called the *remainder.*

47. How do you divide integers when the divisor does not exceed 12?

By setting down only the quotient figures below the dividend.

48. How do you divide when the divisor is above 12?

If it is an incomposite number, I divide the long way, by setting down the operation at full length.

49. Tell me how you find the quotient figures in long division?

An easy method of finding the quotient figures is to see how often the highest figure of the divisor is contained in the highest figure or figures of the dividend; this will produce the first quotient figure.

50. After you find the quotient figure, what is to be done next?

I multiply the divisor by it, and subtract the product from that portion of the dividend which I am dividing.

51. But, if the figure next to the highest in the divisor be 3, 4, or 5, or above 5, it may happen that your quotient figure may be 1, 2, or even 3 beyond what it ought to be, how do you discover this?

If I find the product of the quotient figure multiplied by the divisor, greater than the part of the dividend which I am dividing, I must *rub out* my quotient figure, and take it 1 or 2 less.

52. Can your remainder ever be equal to, or exceed your divisor?

The highest remainder that can possibly happen is 1 less than the divisor; so that if I should in mistake, have a remainder that equals, or exceeds my divisor, I must make my last quotient figure 1 more.

53. After you have found the first quotient figure, how do you proceed for the others?

To the remainder I annex the next figure of the dividend, and afterwards find the other quotient figures precisely as I found the first.

54. How do you divide when the divisor is a composite number?

By dividing by its component parts.

55. How do you find the true remainder when you divide by *two* component parts?

When I divide by two component parts, I multiply the last remainder by the first divisor, adding the first remainder (if any) to the product; this will give the true remainder.

56. How do you find the true remainder when you divide by *three* component parts?

I multiply the second remainder by the first divisor, adding in the first remainder (if any); to this I add the product of the last remainder multiplied by the 1st. and 2nd. divisors; this sum will produce the true remainder.

57. How do you divide by 10, 100, 1000, &c.?

To divide any number by 10 is but reducing it a step lower in the scale of numbers; and, as there is nothing lower than units, the units' place, must, therefore, be the remainder, and the other places the quotient; to divide by 100 the units and tens are the remainder, and the other figures the quotient: and to divide by 1000, &c. the units, tens, and hundreds are the remainder, and the other figures the quotient, &c.

58. How do you divide by any integer that has one, two, three, or more ciphers annexed to it?

By marking off as many figures on the right hand of the dividend as there are ciphers in the divisor, and afterwards dividing by the significant figure or figures in the usual way.

59. How do you divide compound numbers?

I set down in the highest place the number of times that the divisor is contained in it; and what is over I reduce to the next lower name, adding the number of that name (if any); this I divide as before, and so on till the division is completed.

60. How do you divide when there is a fraction annexed to the divisor?

I multiply both divisor and dividend by the under figure of the fraction, and afterwards divide in the usual way.

61. How do you find the proper remainder in this case?

By dividing what is over by the under figure of the fraction.

62. How do you prove Division?

By multiplying the quotient by the divisor, and adding the remainder, (if any) to the product; the sum will be equal to the dividend.

63. Have you any other method of proof?

If I subtract the remainder from the dividend, and divide what is left by the quotient; the result will be equal to the former divisor.

64. Can you produce any other method of proof?

If I add together the remainder and all the products of the several quotient figures according to the order in which they stand in the work; the sum will be equal to the dividend.

VIII. REDUCTION.

65. What is Reduction?

It is that computation by which money, weights, and measures are converted from one denómination to another, without altering their value.

66. How are the various denominations of money, weights, and measures brought from one name or denomination to another?

Three different ways, viz.—1st. by Multiplication; 2ndly. by Division; and 3rdly, by Multiplication and Division.

67. In what case is Multiplication used?

In reducing from a higher to a lower denomination; as from pounds to pence; or from shillings to farthings, &c.

68. When is Division used?

When small names are brought into great; as shillings to pounds; or farthings to pence, &c.

69. When are Multiplication and Division used?

When money, weight, or measure is changed from one denomination to another, equivalent in value ; as to reduce pounds to guineas ; or guineas to pounds.

70. How do you reduce pounds to farthings?

I first bring pounds to shillings by multiplying by 20 ; and shillings multiplied by 12 produce pence ; and pence multiplied by 4 produce farthings :—this is called *Reduction descending*.

71. How do you bring farthings to pounds?

Farthings are brought to pence by dividing by 4 ; and pence are brought to shillings by dividing by 12 ; and shillings are brought to pounds by dividing by 20 :—this is called *Reduction ascending*.

72. How do you bring guineas to pounds?

I may add one twentieth part of the given number to itself, or I may multiply the guineas by 21, to bring them to shillings ; and shillings divided by 20 produce pounds :—this is called *mixt Reduction.*

73. How do you bring pounds to guineas?

From the given number I subtract the one twenty-first part ; the remainder is the guineas ; or, I may multiply by 20, and divide by 21.

74. How do you bring cwts. to qrs. ℔s. oz. and drs.?

I multiply by 4, 28, 16, and 16.

75. How do you bring drs. to oz. ℔s. qrs. and cwts. ?

By dividing by 16, 16, 28, and 4.

76. How do you bring yds. to qrs. na. and inches?

By multiplying by 4, 4, and $2\frac{1}{4}$.

77. How do you bring inches to na. qrs. and yds?

I divide by $2\frac{1}{4}$, 4, and 4. [See 60.]

78. How do you bring yards to Eng. ells?

I multiply yds. by 4, to bring them to qrs. and qrs. divided by 5, produce Eng. ells.

79. How do you bring Eng. ells to yds?

Eng. ells multiplied by 5, produce qrs. and qrs. divided by 4, give yds.

80. Have you any other way of bringing Eng. ells to yds. and yds. to Eng. ells?

If, to the number of Eng. ells, I add $\frac{1}{4}$th. the sum will produce yds. and if, from the given number of yds. I subtract $\frac{1}{5}$th. the result will give the number of Eng. ells.

81. How do you bring Flemish ells to French ells?

By dividing by 2.

82. How do you bring French ells to Flemish ells?

By multiplying by 2.

Note. *Many more queries in Reduction might be produced; but, if the pupil thoroughly understand those that are given, it precludes the necessity of adding to their number.*

IX. ARITHMETICAL FRACTIONS.

83. What is a Fraction?

A Fraction is part or parts of a unit; or of any integer, or whole.

84. How is a Fraction expressed?

By two numbers, one above, and the other below a horizontal line, as $\frac{3}{4}$, $\frac{7}{8}$, &c.

85. What is the number below the line called?

The lower number is called the denominator, and it shows into how many parts the unit, integer, or whole is divided.

86. What is the number above the line called?

It is called the numerator, and shows the number of parts of the unit which the fraction contains.

87. Explain the last two queries by an example.

If I take the fraction $\frac{3}{4}$, the denominator 4 indicates that the unit is divided into 4 equal parts; and the numerator 3 shows that 3 of those parts are to be taken for the value of the fraction.

88. How many kinds of fractions are there?

Four; *proper, improper, compound, and complex.*

89. What is a proper fraction?

It is a fraction whose value is less than unity, and has always the numerator less than the denominator.

90. When is a fraction equal to unity?

When the numerator and denominator are equal.

91. What is an improper fraction?

It is one that is either equal to, or greater than unity; and is known by its numerator being either equal to, or greater than its denominator; as $\frac{4}{4} = 1$, or $\frac{5}{4}$ which is greater than unity.

92. What is a compound fraction?

It is the fraction of a fraction; and is known by two, or more fractions joined together by the particle of; as $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{7}{8}$ &c.

93. What are complex fractions?

They are such as have a fraction in either or both of their members; they arise from operations in Division, where the divisor or dividend or both are mixed numbers: $\frac{3\frac{1}{4}}{5}$, $\frac{7}{4\frac{3}{8}}$, $\frac{\frac{3}{8}}{9}$, $\frac{7\frac{3}{4}}{2\frac{1}{3}}$ are fractions of this kind.

94. What is a mixed number?

A mixed number is composed of a whole number and a fraction; as $3\frac{1}{3}$, $4\frac{5}{8}$, &c.

95. How do you express an integer fractionally?

By placing 1 below it for a denominator.

X. REDUCTION OF FRACTIONS.

96. What is Reduction?

It is the bringing of fractions from one form into another, in order to prepare them for the several operations in Addition, Subtraction, Multiplication, and Division.

97. What is the first case in Reduction?

The first case is to reduce mixed numbers to improper fractions.

98. What is the rule?

I multiply the integer by the denominator of the fraction, and, to the product add the numerator; this sum, written above the denominator, will give the fraction required.

99. Will you explain this?

If I multiply any given number by 4, (which may be supposed to be the denominator of a fraction), it is evident that it thereby becomes 4 times its given value; and if I should make this product the numerator of a fraction, and 4 the denominator, it is obvious that this fraction will just be equal to what the number was originally; for, if any quantity be multiplied and divided by the same number, it is also evident that the quotient must be the same as the quantity first given—hence the rule is manifest.

100. What is the second case?

The second case is to reduce improper fractions to whole or mixed numbers.

101. What is the rule?

I divide the numerator by the denominator, and the result gives the whole, or mixed number.

102, Will you explain this?

Suppose I had $\frac{8}{3}$ to bring to a mixed number; I should then divide 8 by 3, the result is 2, and 2 of a remainder; the mixed number is, therefore, 2 and $\frac{2}{3}$; for, as there are three thirds in 1, the number of times, therefore, that 3 is contained in 8 thirds must, of course, give the number of ones, and the remainder the number of thirds.

103. What is the third case?

The third case is to reduce fractions to their lowest terms.

104. What is the rule?

I divide the greater term by the less, and the divisor by the remainder continually, till nothing remain; the last divisor is the greatest common measure; by which I divide the numerator and denominator of the given fraction.

105. What is the greatest common measure?

It is the greatest number that can possibly be found to divide two, or more numbers without a remainder; those numbers are, therefore, said to be commensurable.

106. How do you find the greatest common measure when there are more than two numbers?

I find the greatest common measure of two of them as before; and of that common measure, and one of the other numbers; and so on, through all the numbers, till the last; then will the last found common measure be the answer.

Note 1. *If* 1 *happen to be the greatest common measure, the given numbers are prime to each other; and are, therefore, said to be incommensurable.*

2. *When the numerator and denominator end with an even number, or a cipher, the fraction can be measured by* 2.

3. *When the numerator and denominator end with* 5, *or* 0, *the fraction is divisible by* 5; *and when they both end with a cipher, or ciphers, the fraction is divisible by* 10, 100, 1000, *&c.*

4. *If two right-hand figures of the numerator and denominator be divisible by* 4, *the fraction is divisible by* 4; *and if three right-hand figures be divisible by* 8, *the fraction is divisible by* 8 *also.*

107. What is the fourth case?

The fourth case is to reduce a compound fraction to a simple one.

108. What is the rule?

I multiply all the numerators of the compound fraction together for the numerator of the simple fraction; and likewise all the denominators for the denominator of the simple fraction; this fraction reduced to its lowest terms will give the fraction required.

109. How can a compound fraction, or a fraction of a fraction be represented by a simple one?

That a compound fraction can be represented by a simple one is evident, since a part of a part must be equal to some part of the whole.

110. Explain this to me.

Suppose I had to bring $\frac{2}{3}$ of $\frac{3}{4}$ to a simple fraction; by the rule, I would say $2 \times 3 = 6$ the numerator, and $3 \times 4 = 12$ the denominator of the required fraction; and $\frac{6}{12}$ brought to its lowest terms is just $\frac{1}{2}$, which is the simple fraction required.

But, should I have to find $\frac{2}{3}$ of $\frac{3}{4}$ of a yard of cloth, it is evident that $\frac{1}{3}$ of 3 qrs. is just 1 qr.; therefore $\frac{2}{3}$ of 3 qrs. are equal to twice one qr. which is $\frac{1}{2}$ yard; this proves the truth of the rule.

Note. *Numbers that are common to both numerator and denominator may be cancelled; thus, in reducing the compound fraction* $\frac{2}{3}$ *of* $\frac{3}{4}$ *of* $\frac{4}{5}$ *of* $\frac{5}{6}$ *to a simple fraction,* 3, 4, *and* 5 *in both numerator and denominator may be cancelled; so that I have* 2 *for the numerator of the required fraction; and* 6 *for the denominator, which is* $\frac{2}{6}$ *or* $\frac{1}{3}$; *this produces the same result as if I had gone through all the process of the rule.*

111. What is the fifth case?

The fifth case is to reduce fractions to a common denominator.

112. How is this done?

By multiplying each numerator by all the denominators, except its own, for a new numerator; and all the denominators together for a new denominator.

113. What effect does reducing fractions to a common denominator produce?

It changes the expression of the fractions without altering their value.

114. But prove to me that the value is not changed as well as the expression of the fraction.

Let me reduce $\frac{3}{4}$ and $\frac{7}{8}$ to equivalent fractions having a common denominator; then, agreeably to the rule, I say,—

$$\frac{3}{4} \times \frac{8}{8} = \frac{24}{32}, \text{ and } \frac{7}{4} \times \frac{4}{8} = \frac{28 \text{ numerators.}}{32 \text{ denominator.}}$$

By placing the numbers properly under one another, in the above manner, it is evident that the numerator and denominator of each fraction are multiplied by the same number; and when there are more than two fractions, by the component parts of the same number; consequently their value is not altered. [See 132.]

115. Why do we bring fractions to a common denominator?

Because, when they have different denominators, they are entirely dissimilar; and, therefore, cannot be incorporated with each other; we can neither add, nor subtract fractions of different denominations with any degree of facility.

In the above example, I cannot easily add $\frac{3}{4}$ and $\frac{7}{8}$ together; but there is no difficulty whatever in adding their equivalents, $\frac{24}{32}$ and $\frac{28}{32}$.

116. What is the sixth case?

It is to reduce a whole number to an equivalent fraction, having a given denominator.

117. What is the rule?

I multiply the whole number by the given denominator; and place the product over the said denominator; this will form the fraction required.

118. Explain this to me.

Suppose I had to reduce 13 to a fraction, whose denominator shall be 12, I should multiply 13 by 12, which produces 156, the numerator of the required fraction; and make 12 the denominator, the fraction then is $\frac{156}{12}$; and as Multiplication and Division are here equally used, consequently, the result is the same as the quantity first proposed; for 156, divided by 12, produces 13.

119. How do you reduce one fraction to another of the same value, having the common denominator assigned?

I multiply the numerator by the assigned denominator, then divide the product by the denominator, the quotient is the numerator, which I place over the assigned denominator.

120. Will you explain this?

If I reduce $\frac{3}{8}$ to another fraction whose denominator shall be 16, I multiply 3 by 16, which produces 48, and 48, divided by 8, gives 6, which is the numerator; the answer then is 32 $\frac{6}{16}$.

121. What is the seventh case?

The seventh case is to bring lower denominations to fractions of higher.

122. What is the rule?

I make the given number the numerator of the fraction, and as many of this name as make one of the higher for a denominator.

Note. *When the given quantity is in more denominations than one, I reduce both it and the higher denomination to one name.*

This rule is so simple that it requires no demonstration; being similar to Reduction of integers, where we have to bring lower names to higher ones.

123. What is the eighth case?

The eighth case is to find the value of fractions.

124. What is the rule?

I reduce the numerator to the next lower name, and divide by the denominator; if there be a remainder, I reduce it to the next lower denomination, and divide as before, and so on till I come to the lowest name.

This being but the division of a compound number, it requires no explanation in this place.

125. What is the ninth case?

It is to reduce fractions from one denomination to another, without altering their value.

126. What is the rule?

If I have to reduce from a higher name to a lower, I multiply the numerator by as many of the less as make one of the greater; but, if from a less to a greater, I multiply the denominator.

The reason of this will be explained in Multiplication and Division.

XI. ADDITION OF FRACTIONS.

127. What is the rule for adding fractions?

I reduce compound fractions to simple ones, and fractions of different denominators to those of the same; then the sum of the numerators written over the common denominator will be the sum of the fractions, which may be valued or reduced, as occasion requires.

Note. *When there is a mixed number, the fraction may be treated the same as the others; and, when the sum of all the fractions is found, the integer may be added afterwards.*

XII. SUBTRACTION OF FRACTIONS.

128. How are the sums in subtraction managed?

The fractions are prepared as in Addition, and the difference set over the common denominator will be the remainder required.

XIII. MULTIPLICATION OF FRACTIONS.

129. What is the rule in Multiplication?

I multiply all the numerators together for the numerator of the product, and all the denominators together for its denominator.

Note. *In Multiplication and Division I must reduce integers and mixed numbers to improper fractions.*

130. What does Multiplication by a fraction imply?

It implies the taking some part or parts of the multiplicand; and, therefore, may very properly be expressed by a compound fraction.

131. Explain this to me.

If I multiply $\frac{3}{4}$ by $\frac{5}{8}$, the result is 15 for the numerator of the required fraction, and 32 for the denominator, which is $\frac{15}{32}$; and the compound fraction $\frac{3}{4}$ of $\frac{5}{8}$ is just equal to $\frac{15}{32}$, as before. [See 108.]

Note 1. *A fraction is best multiplied by an integer by dividing the denominator by it; but, if it will not exactly divide, then I may multiply the numerator by it.*

2. *When the multiplier is 1, the product is the multiplicand; but, when it is a simple fraction, the product must be less than the multiplicand; thus* $8 \times 1 = 8$; *but* $\frac{8}{1} \times \frac{3}{4} = \frac{24}{4} = 6$.

132. Is the value of a fraction altered by multiplying both numerator and denominator by the same number?

No; for the numerator and denominator of $\frac{1}{2}$, multiplied by $2 = \frac{2}{4}$, and $\frac{2}{4} = \frac{1}{2}$. [See 99.]

XIV. DIVISION OF FRACTIONS.

133. What is the rule in Division?

I prepare the fractions as before; then I invert the divisor, and afterwards proceed precisely as in Multiplication: or, I may divide the numerator of the dividend by the numerator of the divisor, and the denominator of the dividend by the denominator of the divisor, if they will divide exactly.

134. Explain this.

If I had to divide $\frac{15}{16}$ by $\frac{3}{4}$, I should invert the divisor $\frac{3}{4}$ and call it $\frac{4}{3}$; then, I should afterwards multiply $\frac{15}{16}$ by $\frac{4}{3}$, which will produce $\frac{60}{48} = \frac{5}{4}$; or, I should divide the dividend $\frac{15}{16}$ by the divisor $\frac{3}{4}$, agreeably to the second part of the rule, which will just give 5 for the numerator of the quotient, and 4 for the denominator, as before,

Note 1. *Division being the reverse of Multiplication, the reason of the rule is evident.*

2. *When the divisor is* 1, *the quotient is the dividend; but, when the divisor is a simple fraction, the quotient will be greater than the dividend; thus,* $8 \div 1 = 8$; *but* $\frac{8}{1} \div \frac{3}{4} = \frac{32}{3} = 10\frac{2}{3}$.

135. Is the value of a fraction altered by dividing both numerator and denominator by the same number?

No; for the numerator and denominator of $\frac{2}{4}$ divided by $2 = \frac{1}{2}$, and $\frac{1}{2} = \frac{2}{4}$.

XV. PROPORTION OF FRACTIONS.*

136. What is the rule in Proportion?

I must state the terms precisely as I did in integers, and afterwards multiply and divide as has been already directed.

---

* This section may be omitted by the pupil till he has obtained a knowledge of the same rule in integers.

137. Produce the solution of the following question :—

The height of my brother William from the ground is $3\frac{1}{4}$ feet, and he casts a shadow of $5\frac{1}{2}$ feet, what is the height of that tree which throws a shadow of $150\frac{1}{2}$ feet?

I should state it in the usual way by saying,

$$\begin{array}{ccccccc} \textit{shad.} & & \textit{shad.} & & \textit{real ht.} & & \\ 5\frac{1}{2} & : & 150\frac{1}{2} & :: & 3\frac{1}{4} & : & 88\frac{41}{44}\ \textit{true height.} \\ 2 & & 2 & & 4 & & \\ \hline \frac{11}{2} & & \frac{301}{2} & & \frac{13}{4} & & \end{array}$$

After producing the solution thus far, I should next invert the divisor (the first term) and proceed as in Multiplication; in the following manner:—$\frac{2}{11} \times \frac{301}{2} \times \frac{13}{4} = \frac{7826}{88} = 88\frac{41}{44}$ feet, the real height of the tree.

XVI. DECIMAL FRACTIONS.‡

138. What is a Decimal fraction?

It is a fraction, the denominator of which may be 10, 100, 1000, &c. or, the denomina-

‡ The famous John Muller, of Konigsberg, in Franconia, better known by the name of Joannes Regiomontanus (from the Latin name of his natal place) is supposed to have been the inventor of Decimal Fractions. His treatise appeared in the year 1464.

tor is unity prefixed to one or more ciphers; as $5 = \frac{5}{10}$; or $.25 = \frac{25}{100}$; or $.127 = \frac{127}{1000}$, &c.

139. How are Decimal Fractions distinguished from integers?

By having a period prefixed to them, which is called the decimal point.

140. How do you read the value of Decimal Fractions?

As the decimal given is the numerator, and the denominator is unity with as many ciphers annexed to it as there are places to the right of the point; I say $.275 = \frac{275}{1000}$, and $.3961 = \frac{3961}{10000}$, &c.

141. What effect has *annexing* ciphers to Decimals?

Ciphers annexed to Decimals make no alteration whatever in their value; for, for every cipher annexed to a decimal, one must be annexed to its denominator; thus .7; .70; .700, &c. are equal to their equivalent arithmetical fractions, $\frac{7}{10}$; $\frac{70}{100}$; $\frac{700}{1000}$; which, it is evident are each equal to the same thing, and, consequently, equal to one another. [See 132.]

142. What effect is produced by *prefixing* ciphers to Decimals?

Prefixing a cipher to Decimals decreases their value in a ten-fold proportion, and two ciphers prefixed decrease their value in a hundred-fold proportion, &c. thus, $.5 = \frac{5}{10}$; but $.05 = \frac{5}{100}$, &c.

XVII. ADDITION OF DECIMALS.

143. How do you set down Decimals to be added?

I place the Decimals in such order that the points shall stand exactly underneath one another.

144. How do you add them?

Precisely as I do integers, observing to make the decimal point in the sum directly under the others.

XVIII. SUBTRACTION OF DECIMALS.

145. How do you place Decimals to be subtracted?

I set the less under the greater in the order directed in Addition; and subtract as in integers, making the decimal point in the difference directly below the others.

XIX. MULTIPLICATION OF DECIMALS.

146. How do you proceed in Multiplication?

Precisely as I do in integers, observing to mark off as many places of the product, beginning at the right hand, as there are places in both multiplier and multiplicand.

Note 1. *When there are not so many places in the product as are necessary to be pointed off, ciphers must be prefixed to make up the deficiency.*

2. *To multiply decimals by* 10, 100, *or* 1000, *&c. is but to remove the decimal point one, two, or three, &c. places towards the right hand.*

147. Will you give me an example?

If I multiply .05 by .04, the result is 20; but, as there are two decimal places in the multiplier, and two in the multiplicand, I must, therefore, prefix two ciphers to 20 to give the proper product; the result, consequently, is .0020.

148. Prove the truth of this.

As $.05 = \frac{5}{100}$, and $.04 = \frac{4}{100}$, then $\frac{5}{100} \times \frac{4}{100} = \frac{20}{10000}$, and $\frac{20}{10000} = .0020$, as before.

149. How do you contract the operation so as to retain so many decimal places in the product as may be thought necessary? ‡

‡ By retaining less than *five* decimal places in the product, the result generally falls short of its proper value.

1st. I set the units' place† of the multiplier under that place of the multiplicand which I wish to retain, and I dispose of the other figures of the multiplier in the contrary order to which they would naturally stand.

2ndly. In finding the product I reject all the figures to the right of the figures by which I am multiplying, and I set down the several products in such order that their right-hand figure may stand directly under one another, observing to increase the first figure of each line by 1, if the product of the figure to the right of the multiplier amount to from 5 to 15; by 2, if from 15 to 25; and by 3 if from 25 to 35, &c.

EXAMPLE.

Multiply 764.32985 by 6.945 so as to retain but five places of Decimals.

Common way.

```
    764.32985
        6.945
    ---------
    382164925
   305731940
  687896865
 458597910
-------------
5308.27080825 Ans.
```

† If the highest place of the multiplier happen to be a small number, it may be necessary to set the units' place one figure farther to the right.

By Contraction.

764.32985	
549.6	
458597910	
68789687	
3057319	
382165	
5308.27081	*Ans.*

XX. DIVISION OF DECIMALS.

150. How do you proceed in Division?

I divide as in integers, and point off as many places for the decimal as the decimal places in the dividend exceed those of the divisor.

Note 1. *When the places of the quotient are not so many as the rule requires, the defect must be supplied by prefixing ciphers.*

2. *When there happens to be a remainder after the division, or, when the decimal places in the divisor exceed those in the dividend, then ciphers may be annexed to the dividend, and the quotient carried on as far as may be thought necessary.*

3. *To divide by* 10, 100, 1000, *&c. is but to remove the decimal point one, two, three, &c. places towards the left hand.*

151. How do you calculate by contraction in Division?

I first determine, by inspection, the value of the first quotient figure, and give it its place in the quotient accordingly; to the product of this first figure I annex a cipher, and subtract as in Division of integers; to find the other figures I divide the last remainder by the divisor, omitting a right-hand figure at each succeeding division, till all the figures in the divisor be exhausted.

Note. *The carriage of the figure omitted must be added to the several products as in Multiplication.*

EXAMPLE.

Divide 6974.32 by 24.798

Common way.

```
24.798)6974.32(281.245 Ans.
       49596
       ------
       201472
       198384
       ------
         30880
         24798
         ------
          60820
          49596
          ------
          112240
            99192
           ------
           130480
           123990
           ------
             6490
             ====
```

By Contraction.

24.798	6974.32(281.245 *Ans.*
	495960
	201472
	198384
2479	3088
	2480
247	608
	496
24	112
	99
2	13
	12
	1

152. Give me the reason of the rule in Division.

The reason of this rule is evident; for, since the divisor multiplied by the quotient gives the dividend, (see 62.) it then follows, from the nature of Multiplication, that the number of places in the dividend are equal to those in the divisor & quotient taken together; consequently, the quotient itself must contain as many places as the places in the dividend exceed those in the divisor.

XXI. REDUCTION OF DECIMALS.

153. How do you reduce an arithmetical fraction to a decimal one?

I divide the numerator by the denominator, annexing ciphers till there be no remainder; or, till the quotient repeat; or, till I have a sufficient number of decimal places.§

154. How do you place the decimal point in this case?

When the division is finished, I point off as many decimal places on the right of the quotient as there were ciphers annexed; and when the quotient has not figures enough for that purpose, I prefix ciphers to make up the deficiency.

155. Give me an example.

Let the given arithmetical fraction whose decimal expression is wanted be $\frac{5}{8}$; now, since every decimal fraction has either 10, 100, or 1000, &c. for the denominator, and, if two fractions are equal, it will be as the denominator of the one is to the denominator of the other, so is the numerator of the given fraction to the numerator wanted; therefore, 8 : 100, &c. ::

$5 : \left(\frac{100 \times 5}{8} = \frac{500}{8} = \right)$ .625 the numerator

of the decimal required, and is the same as by the ule.

156. How do you reduce a decimal to an arithmetical fraction?

I place the decimal for the numerator of the required fraction, with the proper denominator under it, then I reduce the fraction to its lowest terms.

§ Three or four places of Decimals generally answer every purpose in common calculation.

157. How do you reduce integers or decimals to equivalent decimals of higher value?

I divide the integer or decimal by the number of parts in the next higher denomination; and continue the operation to as many higher denominations as may be necessary.

158. How do you proceed when there are several denominations to be reduced to a decimal of the highest?

I set the given denominations perpendicularly under each other for dividends, proceeding from the least to the greatest; then, opposite to each dividend, on the left hand, I place such a number for a divisor as will bring it to the next higher name, and draw a line between them.

159. What is the next thing necessary to be done?

I begin at the uppermost dividend, and perform all the divisions, observing to set each quotient as decimal parts to the dividend on the same line with it; the last quotient will be the decimal required.

160 Explain this by an example.

Let it be required to reduce 15s. $9\frac{3}{4}$d. to the decimal of a pound: $\frac{3}{4}$ths. of a penny brought to a decimal = .75; consequently, $9\frac{3}{4}$ may be expressed by 9.75; but 9.75 = $\frac{975}{100}$ of a penny, or $\frac{975}{1200}$ of a shilling, which, brought to a decimal, gives .8125; therefore, 15s. $9\frac{3}{4}$d. may be expresssed by 15.8125; in like manner, $15.8125 = \frac{158125}{10000}$ of a shilling, which is $\frac{158125}{200000}$ of a pound; and this fraction, brought to a decimal, is £.790625, as by the rule.

161. How do you reduce a decimal to value?

I multiply the given decimal by as many of the next lower name as make one of the given name, observing to mark off as many decimal places in the product as there are places in the given decimal; and, in this manner I proceed till I get to the lowest name; the figures on the left of the decimal points are the several denominations wanted.

162. Will you explain this by an example?

Let me find the value of .785 of a pound; I multiply .785 by 20, (the number of the next lower name that makes one of the name given) the result is 15700; then, as there are three places in the given decimal, I mark off three places in this product, which makes it 15.7 = 15 shillings and 7-tenths of a shilling; then, .7, multiplied by 12 = 8.4 = 8 pence, and 4-tenths of a penny; then, .4, multiplied by 4 = 1.6 = 1 farthing and 6-tenths or 3-fifths of a farthing; the answer is, therefore, 15s. $8\frac{1}{4}$d.

163. How do you find the decimal of any given number of shillings, pence, and farthings, by inspection?

I put down half the greatest even number of shillings for the first decimal figure, and let the farthings in the given pence and farthings possess the second and third places; observing to increase the second place by 5, if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2, when they exceed 37.

164. Give me an example.

Let me reduce 15s. 8½d. to the decimal of a pound:—I first set down .7, which is half of the greatest even number to 15, for the first decimal place; and, as 15 is an odd number, I put 5 in the second place, which gives me .75; in 8½d. there are 34 farthings, and 34 being greater than 12, I increase it by 1, which makes it 35; this occupies the second and third places; the full decimal is, therefore, $.75 + .035 = .785$, the decimal required.

165. How was this rule invented?

The invention of it is as follows:—1st. As shillings are so many 20ths. of a pound, half of any even number of shillings will give me so many 10ths. these, consequently, take the place of 10ths. in the decimal; but, when the number is odd, it will always consist of two figures, the first of which will be half of the greatest even number of shillings, and the second will always be 5; this confirms the rule as far as it respects shillings.

2ndly. Farthings are so many 960ths. of a pound; but, were there 1000 instead of 960 farthings in a pound, it is plain that any number of farthings would just make so many thousandths, and might take their place in the decimal accordingly.

3dly. But, 960, increased by $\frac{1}{24th.}$ part of itself, is 1000; consequently, any number of farthings, increased by their $\frac{1}{24th.}$ part, will give an exact decimal expression for them;—whence, if the number of farthings be more than 12, a $\frac{1}{24th.}$ of the farthings must be greater than a half, and, therefore, 1 must be added; and when the number of farthings is more than 37, a $\frac{1}{24th.}$ part is greater than $1\frac{1}{2}$, for which 2 must be added; and thus the rule is shown to be right.

166. How do you find the value of any decimal part of a pound, by inspection?

As this is the reverse of the preceding case, I double the first figure, or place of 10ths. for shillings; and, if the second be 5 or more, I reckon another shilling, then call the figures in the second and third places (after 5 is deducted) so many farthings, abating 1 when they amount to more than 12, and 2 when they amount to more than 37; the sum of the several results will give the answer.

167. Produce an example.

Let me find the value of .785 of a pound;—I double the place of 10ths. (.7) which gives me 14s. and, as the second place is above 5, I reckon 1s. more, and after the 5 is deducted from the second place, there remains 35 farthings; but, as 35 is above 12, I reckon 1 less, which is 34, and 34 farthings = $8\frac{1}{2}$d. ∴ 14s. + 1s. + $8\frac{1}{2}$d. = 15s. $8\frac{1}{2}$d. the answer.

XXII. PROPORTION.*

168. How are the solutions of the questions in Proportion to be managed?

Precisely as in the same rule in integers, observing that, when fractions or mixt denominations occur in any of the terms, they must be reduced to decimals.

XXIII. SIMPLE PROPORTION, OR, THE RULE OF THREE.

169. What is Simple Proportion, or, the Rule of Three?

It is that rule by which a number is found having to a given number the same ratio which is between two other given numbers.

170. How do you distinguish ratio from proportion?

Ratio is merely the degree in which one quantity exceeds another, or is exceeded by it: so that ratio exists between two quantities or terms; but, in proportion, there must be, at

* The pupil may omit this rule till he has been initiated in the following section.

least, two equal ratios, and consequently, four quantities or terms are necessary to express proportion.

171. How is the measure or quantity of a ratio discovered?

By considering what part or parts the first term is of the second, or, the second of the first, which is known by dividing the one by the other, the quotient expresses the quantity of the ratio.

172. Explain this by an example.

Let the two quantities whose ratio is to be ascertained, be 3 and 9; the ratio of 3 to 9 is 3, since 3 is contained in 9 three times.

In like manner the ratio of 9 to 3 is $\frac{1}{3}$, since 3 is the third part of 9.

173. What is the first thing necessary to be done in stating a question in proportion?

I put for the third term of the proportion the given quantity, which is of the same kind of thing with what is sought.

174. What is done next?

I consider, from the nature of the question, whether the answer should be greater or less than the third term; if greater, I place the less of the other two numbers for the first term, and the greater for the second; but, if less, the contrary.

Note. *Those who have learned arithmetic by* *the common systems will perceive by the above rule that the distinction introduced in them between the Rule of Three Direct and Inverse is wholly disregarded; it is a useless distinction calculated only to perplex the learner, and to render a simple subject complicated.*

175. What other steps are necessary?

I bring the first and second terms to the same name, and the third to the lowest name in it; then, I multiply the second and third terms together, and divide the product by the first; this result will give the answer in the same name to which the third term was reduced.

176. Why do you multiply the second and terms together, and divide by the first?

Let me take an example.

If 8 yards of cloth cost £10, what is the price of 24 yards?

If £10 were the price of 1 yard, 24 times £10, or £240 would be the value of 24 yards; but, as £10 are not the value of 1, but of 8 yards, the eighth part of the above value of 24 yards (which is £30) must be the answer.

It is obvious, then, from this demonstration, that the third term must be multiplied by the second, and the result divided by the first.

177. Why do you bring the first and second terms into the same name, or denomination?

Because, the comparative relation between them cannot be easily seen without it.

178. Will you explain this?

Let me suppose the first term to be 13s. 4d. and the second £4. 6s. 8d. here I do not clearly see the proportion between 13s. 4d. and £4. 6s. 8d. but, if I bring them both into four-pences, I then can see very distinctly the relation they bear to each other, since the one is 40 four-pences, and the other 260.

179. Is it always necessary to bring the third term to the lowest name mentioned in it?

No; for, if the second term be a composite number, it is better to multiply the third term by its component parts without reducing the third.

180. Cannot the labour of multiplying and dividing be materially shortened;

The work may be very much abridged, if I can find a number that will divide the first and second terms without a remainder, and use the quotients instead of the original terms.

181. Prove that the quotients in the following question bear the same proportion to one another that the original terms do.

What is the value of 24 yards of cloth at the rate of £5 for 6 yards.

yds. *yds.* £
6 : 24 :: 5
5
6)120
£20

or

yds. *yds.* £
6 : 24 :: 5
1 4
5
£20

As 6 will divide the first and second terms without a remainder, I have employed this number, as it is the greatest that can be found for that purpose.

Here it is evident that using 1 and 4 (the quotients) produces the same result as 6 and 24, the original terms do; for 1 bears the same proportion to four times 1, or 4, that 6 bears to four times 6, or 24; or, since the ratio that exists between 6 and 24 is the same as the ratio between 1 and 4, the results must be the same.

XXIV. COMPOUND PROPORTION.

182. What is Compound Proportion?

Compound Proportion is that rule which has one number of the same kind of thing with what is wanted for the answer; and it may have four, six, eight, or ten, &c. numbers besides, to find a fifth, seventh, ninth, or eleventh, &c.

183. What is the first thing necessary to be done in solving questions in this rule?

I first write down that number for the third term which is of the same kind of thing with what is wanted for the answer, as in the simple rule.

184. What is to be done next?

I then take two numbers of the same kind, and place them as I did in Simple Proportion; and, afterwards, I place the remaining pairs of the same kind underneath the first pair.

185. After having stated the question in this manner,—what is to be done next?

I reduce like pairs to the same name, and the third term to the lowest name in it, (if necessary); then I find the product of the numbers that are below one another, which will reduce the terms to three; then I proceed with these three numbers precisely as I did in Simple Proportion.

Note. *If the pupil rightly understand abridgment, he may use it to very great advantage in this rule.*

XXV. PRACTICE.

186. What is Practice?

It is a short and expeditious method of calculating the value of goods by taking aliquot parts.

187. What is an aliquot part?

It is such a part as is contained in an integer an exact number of times; thus, 2, 4, 5, and 10 are aliquot parts of 20.

188. What is an aliquant part?

It is a part of a number which, however repeated, will never make up the number exactly; thus 3 is an aliquant part of 10; three threes being less than 10, and four threes more than 10.

189. What is the rule when the price is less than a penny?

I take the aliquot part or parts that are in a penny, and, afterwards, divide by 12 and 20.

190. Will you explain this?

Let it be required to find the price of 240 yards, at $\frac{1}{2}$ per yard; were the price a penny per yard, the answer would just be as many pence as there are yards; but, as the price is but half of a penny, the answer is, therefore, as many pence as half the number of yards; and pence, divided by 12 & 20, produce pounds.

Note. *In Practice, the result is of the same denomination of which the aliquot parts are taken.*

191. What is the rule when the price is less than a shilling?

I take the aliquot part or parts that are in a shilling, and divide the sum by 20.

192. Will you explain this rule?

If I had to find the value of 240 yards, at a shilling per yard, the answer would be 240 shillings; but, at 6d. per yard, the value will, of course, be as many shillings as half the number of yards; and, shillings, divided by 20, produce pounds.

193. What is the rule when the price is less than a pound?

I take the aliquot part or parts that are in a pound, the sum of which will be the answer.

194. Will you explain this?

It is evident that 240 yards, at a pound per yard, is £240; but, at 10s. (which is half of a pound) per yard, the result will just be the half of £240, which is £120.

195. What is the rule when the price consists of an even number of shillings?

I multiply the quantity by half the price, and double the units' place of the product for shillings.

196. Give me the reason for this.

The answer could be found by multiplying by the number of shillings, and dividing by 20; and, it is evident, from the nature of abridgments, that, multiplying by half the number of shillings, and dividing by 10 (which is the half of 20) will produce the same result; and, when the divisor is 10, double the units' place (or remainder) will always produce the value of the remainder in shillings.*

197. What is the rule when the price is an odd number of shillings?

I find the value at the highest even number of shillings, as I did in the last case; and, afterwards, add one-twentieth part of the top line for the value of the odd shilling.

Note. *When the price is* 15s. *if I subtract one-fourth of the top line from itself, the remainder is the answer; at* 16s. *I subtract one-fifth part; at* 18s. *one-tenth part; and, at* 19s. *one-twentieth part; at* 21s. *I add one-twentieth part; at* 22s. *one-tenth part; at* 24s. *one-fifth part; at* 25s. *one-fourth part, &c.*

198. What is the rule when there is a fraction in the quantity?

I calculate as before, and add a proportional part of the price for the fraction.

199. Is there any other method?

I may annex to the quantity such a part of a pound as the fraction is of a unit; and, afterwards, take parts for the price, as before.

200. What is the rule when the price is pounds, or pounds and shillings, &c.?

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* Or, it is the same as multiplying by the decimal of the price, and valuing the product by inspection. [See end of Sec. xxi.]

I multiply the quantity by the pounds, and take aliquot parts for the other denominations.

201. What is the rule when the quantity is in several denominations?

I first find the value of the highest denomination, and then take aliquot parts for the lower ones.

202. Have you any other method of finding the value of cwts. qrs. and lbs.?

Yes; I may reduce the quantity to the form of pounds, shillings, and pence, by multiplying the lbs. by $2\frac{1}{7}$, reckoning the product pence, and the qrs. by 5, reckoning the product shillings; the cwts. prefixed to this will give the value of the quantity at £1 per cwt. then I multiply this value by the number of pounds in the price, and take aliquot parts for the shillings and pence, as follows:

Suppose I had to find the value of 98 cwt. 3 qrs. 21lbs. at £5. 6s. 8d. per cwt.

	thus,—98	: 3	: 21		
		5	: $2\frac{1}{7}$		
6s. 8d. = $\frac{1}{3}$	£98	: 18	: 9	= value at £1	
			5		
	494	: 13	: 9	= value at £5	
	32	: 19	: 7	= value at 6s. 8d	
	£527	: 13	: 4	*Ans.*	

XXVI. ALLOWANCES ON GOODS.

203. How many deductions are allowed on goods?

Three; Tare, Trett, and Cloff.

204. What is the Gross weight.

It is the weight of the goods and what contains them.

205. What is the tare?

It is the weight of the chest, barrel, or hogshead, &c. which contains the goods.

206. What is Tret?

It is an allowance of 4 lbs. on every 104 lbs. or one-twenty-sixth part on goods liable to waste.

207. What is Cloff?

Cloff is an allowance of 2lbs. on every 3 cwt. or, one-hundred-and-sixty-eighth part given to retailers for the turning of the scales.

208. What is Suttle weight?

Suttle weight is what remains after the Tare is deducted from the Gross.

209. What is Net weight?

It is what remains after the proposed deductions are made.

210. What is the first case in Allowances on Goods?

It is when the Tare is inserted in the invoice along with the Gross weight; or, when it is so much on the whole.

211. How do you find the Net weight in this case.

I subtract the Tare from the Gross, and the remainder will be the Net weight.

212. What is the second case?

It is when the Tare is at a certain rate per cwt.

213. What is the rule for finding the Net weight?

I take aliquot parts of a cwt. for the Tare, and subtract it from the Gross; the remainder will be the Net weight.

214. What is the third case?

It is when there is an allowance both of Tare and Tret.

215. How is the Net weight found in this case?

I deduct the Tare from the Gross, as before directed, and, afterwards, deduct one-twenty-sixth part of the Suttle for the Tret; the last remainder will be the Net.

216. What is the fourth case?

It is when Tare, Tret, and Cloff are allowed.

217. What is the rule for finding the Net weight?

I deduct the Tare and Tret, as before directed; and, afterwards, one-one-hundred-and-sixty-eighth part of the last remainder, which will produce the Net.

XXVII. SIMPLE INTEREST.

218. What is Simple Interest?

It is an allowance given by the borrower to the lender of money, for the use which he has had of it.

219. What is the principal?

It is the money laid out to interest.

220. What is the rate per cent.

It is the sum allowed on every hundred pounds.

221. What is the amount?

The amount is the sum of the principal and interest.

222. What is the rule for finding the interest for any number of years?

I multiply the principal by the rate per cent, and number of years; and this product I divide by 100.

223. Give me another method.

I take aliquot parts of 100 for the rate per cent, which will give the interest for one year; then I multiply this interest by the number of years.

224. Tell me another method.

I may multiply the rate per cent and years together, and take aliquot parts of 100 for the product.

225. Can you produce another method?

Yes; I may say 100 is in the same proportion to the principal that the rate per cent multiplied by the number of years is to the interest required.

226. How do you calculate interest for days?

I multiply the principal by the days, and the result by twice the rate; and then divide by 73000.

227. Will you explain this?

If I first find the interest for one year, (or 365 days) and multiply this interest by the number of days, and divide by 365, it will produce the answer: in performing this I multiply by the rate per cent and the number of days, and I divide by 100 and 365; or, at one division by 36500; and, agreeably to the rule of proportion, if I multiply by twice the rate, I must divide by twice 36500, which is 73000.

228. Have you any other way?

Yes; I may multiply the principal by the rate per cent, and the days and divide by 36500.

229. Do you know any other method?

The usual way is to find the interest of the given sum for a year; then say 365 days are to the given days, as one year's interest is to the interest required.

230. Can you tell me any other method?

I may multiply the principal by the number of days, and divide by 7300 for the interest at 5 per cent; and, from having the interest at 5 per cent, the interest at any other rate may be very easily found.

231. How does dividing by 7300 produce the interest at 5 per cent?

As any principal multiplied by 5 and divided by 100 gives the interest of the same for a year or 365 days at five per cent; so this interest multiplied by the number of days and divided by 365 will produce the answer.

But as 5 a multiplier, and 365 a divisor are both divisible by 5; consequently, the multiplier may be cancelled, and the fifth part of 365 (73) is the divisor, the other divisor is 100; and 73, and 100 are the component parts of 7300.

232. What is the legal interest in Great Britain?

The legal interest is 5 per cent which is one-twentieth of the principal.

233. How do you calculate interest on bills, when partial payments are made at different times?

I multiply the principal and the several balances in the order of their dates by the number of days between the times of payment, and divide the sum of the products by 7300 for the interest at 5 per cent.

234. How do you compute interest on cash accounts, or any other progressive account, when partial payments are made, and partial debts contracted?

In computing interest on accounts current, I add and subtract the sums paid and received in the order of their dates, and, if the balance be sometimes due to one party, and sometimes to the other, I extend the product in different columns, and divide the difference of the sum of the columns by 7300 for the interest at 5 per cent.

235. How do you find the balance when partial payments are made on bonds, or bills, at intervals greater than a year?

The usual way is to add the interest at the times of payment to the principal, and, from that amount, to deduct the payment; the remainder is the balance.

XXVIII. COMPOUND INTEREST.

236. What is Compound Interest?

It is that which arises from both principal and interest taken together, at the end of each stated time when the interest becomes due.

237. How is Compound Interest calculated?

I first find the interest of the principal, for the first stated period, as directed in Simple Interest, and add this interest to the principal; then I find the interest of this amount for the second stated period, and so on for the number of periods, this will produce the amount, and, if I subtract the principal from the amount, the remainder will give the interest.

238. Is there any other method of finding the amount in Compound Interest?

Yes; if I first find the amount of £1 for the time of the first payment, and involve it to the power whose index is denoted by the number of payments, this will give the amount of £1 for the time and rate, which amount, if multiplied by the principal, the product will give the amount required. [See Sec. 48].

Tell me by this method the amount of £700 lent for 8 years, at 5 per cent, Compound Interest.

The amount of £1 for a year is £1.05, and as the number of payments is 8, which is the index of the power to which the amount is to be raised, I involve it in the following manner: viz.

```
      1.05 = 1 year's amount of £1
      1.05
      ----
      5 25
    105
    ------
    1.1025 = 2 years' amount
      5201.1
     11025 0
      1102 5
        22 0
         5 5
    --------
    1.2155 0 = 4 years' amount re-
      5512 1    taining only 5 deci-
    --------    mal places.
    1 2155 0
      2431 0
       121 6
        60 8
         6 1
    --------
    1.4774 5 = 8 years' amount.
        70 0 = The principal.
  ----------
  1033.2150 0 = The amount of £700
or, Ans. £1033 : 4 : 3½  for 8 years.
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239. Can money be lent lawfully at Compound Interest?

No; but in granting or purchasing annuities, leases, or reversions, Compound Interest is allowed.

XXIX. REBATE OR DISCOUNT.

240. What is Rebate or Discount?

It is when a sum of money due at any time to come, is satisfied by paying so much present money, as being put out to interest would amount to the original sum in the same space of time.

241. What is the rule for finding the present worth, or prompt payment of any sum, due at any time hence?

I say as the amount of £100 at the given rate and time is to the sum, so is £100 to the present worth.

242. How do you find the discount?

The difference between any sum and its present worth is the discount, it may also be found as follows:—The amount of £100 at the given rate and time is to the given sum, as the interest of £100 for the same time is to the discount.

243. Is there any other method of finding the discount?

Yes; Bankers and others who keep money for the purpose of discounting bills, add three days of grace to the time the bill has to run, and calculate the discount in the same way in which the interest is found.

Note. *When the discount is taken from the sum of the bill, the balance is, in business, called the* proceeds.

XXX. COMMISSION.

244. What is Commission?

It is an allowance of so much per cent to a factor or correspondent abroad, for buying and selling goods, and negociating bills, &c. for his employer.

245. How is Commission calculated?

Precisely in the same way as I calculate interest.

XXXI. BROKERAGE.

246. What is Brokerage?

It is an allowance made to a broker for assisting merchants or factors in procuring or disposing of goods.

247. How is Brokerage calculated?

In the same way in which interest is calculated.

XXXII. INSURANCE.

248. What is Insurance?

It is a per centage given to certain persons or offices who engage to make good the loss of ships, houses, or merchandize, which may happen to be destroyed by storm, fire, or any other accident.

249. What is the first case?

It is the sum insured, and rate per cent, given to find the premium.†

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† Premium, is the allowance given to the underwriter for engaging to repay losses.

250. How is the premium found?

The same way in which interest is calculated.

251. What is the second case?

It is when the rates per cent, premium, discount, and real value at risk are given to find what sum must be insured to cover the whole outset on a single voyage; that is, to recover from the underwriters the whole value at risk; including premium and discount.

252. What is the rule in this case?

I may either multiply the value of the property by 100, and divide by the difference between 100 and the rate; or,—

I may subtract the sum of the premium and discount from 100, then, as the remainder is to 100, so is the value at risk to the answer.

253. What is the third case?

It is to find how much will cover a certain sum in a voyage out and home.

254. What is the rule in this case?

I first find by the second case what will cover the given sum in the voyage out, and then I find in the same manner what must be insured to cover the sum found in the homeward voyage.

255. What is the fourth case?

It is when the sum insured, and the rate per cent discount are given, to find the short recovery.†

256. What is the rule in this case?

I say as 100 is to 100 minus the discount, so is the sum insured, to the short recovery; or, the short recovery may be found by taking aliquot parts.

XXXIII. BUYING AND SELLING OF STOCK.

257. What is the Buying and Selling of Stock.

It is the purchasing or disposing of a certain sum of money in the Bank of England, or in the capital of some trading company, according to the rate per cent which it may sell for, at any given time.

258. What is the first case?

It is to find the value of any given quantity of stock.

259. What is the rule in this case?

I multiply by the rate and divide by 100.

260. What is the second case?

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† It is usually an article in the policy, that in case of loss, the insurer is to be allowed a small discount, commonly 2 per cent, so that the insured receive only £98 for every £100, and these £98 are called the SHORT RECOVERY.

It is to find the quantity of stock that can be purchased by a given sum.

261. What is the rule in this case?

I multiply by 100 and divide by the rate.

262. What is the third case?

It is to find the rate of interest.

263. What is the rule in this case?

I multiply the interest or dividend by 100, and divide by the rate, or current price.

XXXIV. EQUATION OF PAYMENTS.

264. What is Equation of Payments?

It is the finding a time to pay at once, several debts due at different times, so that no loss may be sustained by either party.

265. What is the rule for finding the proper time for payment all at once?

I multiply each payment by the time at which it is due, then I divide the sum of the products by the sum of the payments; the quotient will give the time required.

XXXV. BARTER.

266. What is Barter?

It is the exchanging of one kind of commodity for another, and directs Merchants and Tradesmen so to proportion the value of their goods that neither party may sustain loss.

267. Give me the rule for calculating questions in Barter?

I first find the value of the commodity whose quantity is given, then I find what quantity of the other, at the rate proposed, may be had for the same money; the result will give the answer.

268. How do you proceed when one has goods at a certain price ready money; but, in Barter, advances it something more.

I find what the other ought to rate his goods at in proportion to that advance, and then I proceed as before.

XXXVI. PROFIT AND LOSS.

269. Explain Profit and Loss.

It is by this rule that Merchants are enabled to make a proper estimate of the different articles of their trade, and instructs them so to raise or lower the price of their commodities as to gain or lose so much per cent on the same.

270. What is the first case?

It is when the prime cost* and profit or loss

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* Prime cost is what is paid for goods when they are purchased.

upon it are given to find the profit or loss per cent.

271. What is the rule in this case?

I say, as the prime cost is to the profit or loss, so is 100 to the profit or loss per cent.

272. What is the second case?

It is when the rate per cent and prime cost are given to find the selling price.

273. What is the rule in this case?

I say, as 100 with the rate per cent, added in case of gain, and deducted in case of loss, is to 100 so is the prime cost to the selling price.

274. What is the third case?

It is when the selling price and rate per cent, profit or loss are given to find the prime cost.

275. What is the rule in this case?

I say, as 100, with the rate added or deducted, is to 100, so is the selling price to the prime cost.

276. What is the fourth case?

It is to find a proportional rate on one advanced price, by having another and the rate on it given.

277. What is the rule in this case?

I say, as the price, whose rate per cent is given, is to 100, with the given rate added or deducted, so is the other given price to a fourth number; from which I subtract 100 in case of gain; but, which I subtract from 100 in case of loss: the result will be the required rate.

278. What is the fifth case?

It is when the whole gain or loss and the rate per cent are given to find what the whole was bought and sold at.

279. What is the rule in this case?

I say, as the rate is to 100, so is the gain to the buying price, and the selling price is found by adding the gain to the buying price, or subtracting the loss from it.

XXXVII. COMPANY, OR PARTNERSHIP.

280. What is Company, or Partnership?

It is when two or more Merchants join their stocks to carry on some concerted branch of business.

281. What is the first case?

It is when all the partners are equally concerned.

282. How are the profits arising from the business divided in this case?

I divide the whole gain by the number of partners; the quotient will give the gain of each.

283. What is the second case?

It is when the Company's capital is divided into a certain number of shares.

284. How is the gain in this case divided?

The whole gain must be divided by the whole number of shares, this will give the amount of one share; this share, multiplied by the number of shares that each partner holds in the concern, will produce the gain of each.

285. What is the third case?

It is when each partner is concerned only to a certain extent.

286. What is the rule in this case for finding each partner's share of gain?

I say, as the total stock is to each partner's stock, so is the total gain to the gain of each.

287. What is the fourth case?

It is when the dividend is to be proportioned to the time the capital is employed.

288. How do you calculate the gain in this case?

I say, as the sum of the products of the capitals into the times they are respectively employed is to the whole dividend, so is each particular product to its dividend.

XXXVIII. BANKRUPTCY.

289. In what case does Bankruptcy happen?

When a person becomes insolvent, or is not able to pay 20s. per pound, of his debts.

290. How are the several dividends in Bankruptcy calculated?

Precisely as the gains in partnership are computed.

XXXIX. EXCHANGE.

291. What is Exchange?

Exchange is the receiving or paying of money in one country for its equivalent in the money of another, negociated by means of bills of Exchange.

292. What is a bill of Exchange?

A piece of paper,* on which is written a short order, for paying at an appointed time, to any person, or his order, a certain sum of money.

293. How are the computations in Exchange performed?

They are generally performed by Proportion, or by Practice.

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* Bills drawn in Britain, whether Inland, or upon Foreign Countries, are illegal, unless written upon paper stamped for the purpose; but, bills on Britain from Foreign Countries are valid without any stamp.

XL. POSITION.

294. What is Position?

Position, or The Rule of False, is that rule which, by the help of supposed numbers, finds a true answer to many intricate questions.

295. Into how many parts is it divided?

Into two; Single and Double Position.

296. What is Single Position?

It is Single Position when there is occasion for one supposition only.

297. What is the rule in this case?

I say, as the result arising from the supposition is to the supposition, so is the given number to the answer.

298. What is Double Position?

It is when a question is so involved, that a simple division cannot be readily adapted to all the conditions of it but by two suppositions.

299. What is the rule in this case?

1st. I take any two convenient numbers, and proceed with each agreeably to the conditions of the questions, and find the differences or errors betwixt the results and the given number.

2ndly. I multiply each of these errors into the other's supposition, and, if both errors be of the same kind, that is, both greater or both less than the given number, I divide the difference of the products by the difference of the errors.

3dly. If the errors be not of the same kind, that is, if the one be greater, and the other less than the given number, I divide the sum of the products by the sum of the errors; the quotient, in either case, will be the answer.

XLI. PURCHASING OF FREEHOLD ESTATES.

300. What are Freehold Estates?

All freehold or real estates are such as are bought to continue for ever.

301. What is the first case?

It is the yearly rent and rate per cent being given to find the price or value of the estate.

302. How do you calculate the value of the estate?

I say, as the rate per cent is to 100, so is the yearly rent to the value of the estate.

303. What is the second case?

It is the value of the estate and rate of interest being given to find the yearly rent.

304. How is the yearly rent calculated?

I say, as 100 is to the rate, so is the value of the estate to the yearly rent.

305, What is the third case?

It is the price of the estate and rent given to find the rate of interest.

306. How do you find the rate of interest?

I say, as the price is to the rent, so is 100 to to the rate per cent.

307. What is the fourth case?

It is when the rate of interest is given to find at how many years' purchase any estate may be bought.

308. How are the number of years found?

I divide 100 by the assigned rate of interest, and the quotient will give the number of years.

309. What is the fifth case?

It is the number of years' purchase at which an estate was bought given to find the rate of interest.

310. How is the rate found?

I divide 100 by the number of years; the quotient will produce the rate.

XLII. PURCHASING OF FREEHOLD ESTATES IN REVERSION.

311. What is the meaning of the word Reversion?

The word Reversion signifies a returning or coming back again, and when applied to the purchasing of estates, means that the possession of the estate is to commence some given number of years hence.

312. What is the first case?

It is the rate of interest, time prior to the commencement of the Reversion, and rent of the estate being given to find the present value of the Reversion.

313. How is the value of the Reversion found?

1st. I find the value of the estate, as if it were to be possessed immediately.

2ndly. I find the present value of the rent for the given number of years, prior to the commencement of the Reversion.

3rdly. I subtract the last found value from the first, and the remainder will give the true value of the Reversion. Or, as the amount of £1 for the time and rate is to £1, so is the estate to the Reversion.

314. What is the second case?

It is the value of a Reversion, the time prior to its commencement, and the rate of interest being given to find the yearly rent.

315. How is the yearly rent found?

I say, as 100 is to the rate of interest, so is the amount of the Reversion, (at the rate and for the time), to the yearly rent.

XLIII. ALLIGATION, OR RULE OF MIXTURES.

316. What is the use of Alligation?

By this rule Apothecaries, Grocers, Spirit-dealers and others, discover the mean rate of a mixture, compounded of divers simples given; and the quantity of simples necessary to make a mixture of an assigned rate or quantity; for which reason, this rule falls under two denominations: Alligation Medial, and Alligation Alternate.

317. What is Alligation Medial?

It is when the price and quantities of several simples are given to find the mean price of that simple.

318. How is the mean price found?

By saying, as the whole composition is to its total value, so is any part of the composition to its mean price.

319. How do you find how much of each simple is in any assigned portion of the mixture?

I say, as the whole quantity of the mixture is to the several quantities of the mixture given, so is the quantity of the assigned portion to the quantities of the simples sought.

320. How is the quantity of a mixture increased or diminished?

In this way—as the sum of the given quantities of the simples is to the several quantities given, so is the quantity of the mixture proposed to the quantity of the simples sought.

321. How are the several quantities mixed

found, from having the total of a mixture, with the whole value, and the value of the several ingredients?

I multiply the total of the mixture by the least value, subtract the product from the total value, and the remainder gives the dividend; I then subtract the value of the lowest rated unit from the value of the highest rated unit, and the remainder gives the first divisor; the quotient arising from these two factors shows the quantity of the highest priced ingredient; and the other is the complement to the whole.

Note. *Questions in this rule may be proved by finding the value of the whole mixture at the mean rate; if it agree with the total value of the several quantities at their respective prices, the work is right.*

322. What is Alligation Alternate?

Alligation Alternate is the method of finding what quantity of each of the simples, whose rates are given, will compose a mixture of a given rate; so that it is the reverse of Alligation Medial, and, may therefore, be proved by it.

323. What is the rule in this case?

1st. I first write the rates of the simples under each other, with the mixture rate on the left-hand.

2ndly. I arrange the rates of the simples so that one less than the mixture rate be always linked to one that is greater.

3rdly. I write the difference betwixt the mixture rate and that of each of the simples opposite to that rate with which it is linked.

4thly. These differences will be the quantities at the rates opposite to which they stand.

Note. *Questions in this rule admit of a great variety of answers; according as they are linked, which may all be found to be true; for, having obtained one answer, we may obtain as many more as we please, by increasing or diminishing the alternate differences of the simples.*

324. What is the rule when the whole composition is limited to a certain quantity?

I find the answer as before, by linking; then, as the sum of the quantities found is to the whole limited quantity, so is each quantity found by linking, to the quantity required.

325. What is the rule when one of the ingredients or simples is limited?

I say, as the quantity of that simple found by linking is to the quantity limited, so are the other quantities found to the required quantities of each.

XLIV. ALTERNATION PARTIAL.

326. What is Alternation Partial?

It is when the prices of all the simples, the quantity of but one of them, and the mean rate are given, to find the several quantities of the rest in proportion to that given.

327. What is the rule in this case?

I first take the difference of each price, and the mean rate as directed in the preceding section; then, as the difference of that simple, whose quantity is given, is to the rest of the differences severally, so is the quantity given to the quantities required.

XLV. ALTERNATION TOTAL.

328. What is Alternation Total?

It is when the price of each simple, the quantities to be compounded, and the mean rate are given to find how much of each sort will make that quantity.

329. What is the rule in this case?

I take the difference between each price and the mean rate as before; then, as the sum of the differences is to each particular difference, so is the given quantity to the quantity required.

XLVI. ARITHMETICAL PROGRESSION.

330. What is meant by Arithmetical Progression?

Any rank or series of numbers increasing or decreasing by a common difference are said to be in Arithmetical Progression.

331. Explain this to me?

The numbers 2, 4, 6, 8, &c. and 8, 6, 4, 2, &c. are in Arithmetical Progression, for they increase and decrease by 2, which is called the common difference.

332. What are the numbers which form the series called?

They are denominated the terms of the Progression.

333. How many terms are there in Arithmetical Progression?

There are five which are as follows:—

1. The first term } commonly called the
2. The last term } extremes.
3. The number of terms
4. The common difference,—and—
5. The sum of all the terms;

and, any three of which being given, the rest may be easily found.

Note 1. *When the terms are even, the sum of the extremes will be equal to the sum of the two means, or, of any two numbers equally distant from the extremes; as,* 2, 4, 6, 8, *where* 2+8, *the extremes are equal to* 4+6, *the means.*

2. *But when the terms are odd, the sum of the extremes is equal to double the means, or, the sum of any two terms equally distant from the means, is just double the means; as,* 2, 4, 6, 8, 10,

where 2+10 = 12 *which is just double of* 6, *the means.*

334. How do you find the common difference, when the extremes and number of terms are given?

I divide the difference of the extremes by the number of terms, less 1; the result will give the common difference.

335. Give me the reason of this rule?

The difference of the first and last terms evidently shows the increase of the first term by all the subsequent additions, till it becomes equal to the last, and, as the number of those additions are evidently 1 less than the number of terms, and the increase by every addition equal; it is plain that the total increase divided by the number of additions must give the difference at every one separately; whence the rule is manifest.

336. How do you find the other extreme, when one of the extremes, the common difference, and the number of terms are given?

I multiply the common difference by the number of terms, less 1, the product will give the difference of the extremes; which, added to the less extreme, will give the greater; or, subtracted from the greater will give the less.

337. How do you find the number of terms, when the extremes and common difference are given?

I divide the difference of the extremes by the common difference, and the quotient, plus 1 will be the number of terms.

338. Explain this rule?

The difference of the extremes divided by the number of terms, less 1, gives the common difference, (See 334) consequently, the same divided by the common difference must give the number of terms, less 1; hence this quotient, augmented by 1, must be the number of terms.

339. How do you find the sum of all the terms, when the extremes and number of terms are given?

1st. I multiply the sum of the extremes by the number of terms and divide by 2; the quotient will be the answer.

2ndly. If I multiply the sum of the extremes by half the number of terms, the product will produce the answer;—and,—

3rdly. If I multiply half the sum of the extremes by the number of terms, the result will produce the answer.

340. How do you find the extremes, when the number of terms, common difference, and sum of all the terms are given?

I divide the sum of the terms by the number of terms, and from the quotient subtract half the product of the common difference multiplied by the number of terms, less 1, and the remainder will give the first term, then I find the other by No. 336.

341. How do you find the arithmetical mean proportion between two given numbers?

I divide the sum of the terms by 2; the quotient gives the answer required.

XLVII. GEOMETRICAL PROGRESSION.

342. What is Geometrical Progression?

It is the increasing or decreasing of any rank of numbers, by some common ratio, as, 2, 4, 8, 16, increasing by the ratio 2; and 16, 8, 4, 2, decreasing by the ratio $\frac{1}{2}$.

343. What are the numbers which form the series called?

They are denominated the terms of the Progression, and are five in number: viz.—

1. The first term } commonly called the
2. The last term } extremes.
3. The number of terms
4. The ratio; or, common multiplier
5. The sum of all the terms.

Note 1. *In any Geometrical Progression of three terms, the square of the mean term is equal to the product of the extremes; thus, in* 2, 6, 18, *it will be* $6^2 = 36$; *and* $2 \times 18 = 36$ *also.*

2. *If the series contain any odd number of terms, the square of the mean will be equal to the*

product of the adjoining extremes; or, of any two equally distant from them.

3. *In any geometrical series of four terms, the product of the two means is equal to the product of the two extremes; thus,* 3 : 6 : : 12 : 24, *where* $6 \times 12 = 72$; *and* $3 \times 24 = 72$ *also.*

Obs. It is plain from the nature of multiplication, that, if one factor be increased in the same ratio in which the other is diminished, their product will still be the same. Hence, in the above series, as 6 exceeds 3 in the same ratio that 24 exceeds 12, it is manifest, that the product of the extremes will always be equal to that of the means.

4. *If the series contain an even number of terms, the product of the means will be equal to the product of the adjoining extremes, or, of any other pair equally distant from them.*

344. How do you find the sum of the series when the first term, the last term, and the ratio are given?

I multiply the last term by the ratio and from the product subtract the first term; the remainder divided by the ratio, less 1, will give the sum of the series.

345. What is the rule for finding any remote term without producing all the intermediate ones, when the ratio is given?

I find what figures of the indices added together will give the index or exponent of the term wanted; then I multiply the numbers standing under each index into each other; this result will produce the term required.

346. When the first term, and the number of terms are given to find the last—what is the rule?

1st. I write down a few of the leading terms in the geometrical series, and, over them, the indices.

2ndly. I add together the most convenient indices to make an index less, by unity, than the number expressing the place of the term sought.

3dly. I multiply together the terms of the geometrical series belonging to those indices which were added, and I make the product a *dividend.*

4thly. I raise the first term to a power whose index is equal to the number, less one, of the term multiplied, and I make the result a *divisor.*

5thly. If I divide the dividend by the divisor, the quotient will be the term sought.

Note. 1. *As the last term in a long series of numbers is very tedious to come at by continual multiplication; in order that it may be more easily found, there is a series of numbers made use of in arithmetical progression, called indices, beginning with a unit, whose common difference is* 1: *whatever number of indices is made use of, I must set the same numbers (in such geometrical* *proportion as is given in the question) under them —as—*

1, 2, 3. 4, 5, 6, *Indices,*

2, 4, 8, 16, 32, 64, *Numbers in geometrical proportion.*

2. *If the first term in the geometrical proportion be different from the ratio, the indices must begin with a cipher—as*

0, 1, 2, 3, 4, 5, 6, *Indices.*

1. 2, 4, 8, 16, 32, 64, *Numbers in geometrical proportion.*

3. *When the indices begin with a cipher, the sum of the indices made choice of must always be* 1 *less than the number of terms given in the question; for,* 1 *in the indices is over the second term, and* 2 *over the third, &c.*

XLVIII. INVOLUTION.

347. What is the nature of Involution?

Involution is the method of raising powers from a given number or root. Thus, if the first power or root be multiplied by itself, the product is the second power or *square;* this again by the root, and the product is the third power or *cube;* and, if this be again multiplied by the root, or the square multiplied by the square, the product is the fourth power, or *biquadrate;* and so on.

348. What is the exponent, or index of a power?

The figure that denotes the extent or degree of that power, and exceeds, by 1, the number of multiplications used in producing the power; so 2 is the exponent or index of the square, 3 the index of the cube, and 4 that of the biquadrate, &c.

349. What is the general rule for finding powers?

I multiply the given number or root continually by itself, till the number of multiplications be 1 less than the index of the power to be found.

XLIX. EVOLUTION.

350. What is the nature of Evolution?

Evolution is the method of extracting the roots of given powers, and is the reverse of *Involution*.

351. Repeat the rule for extracting the square root.

1st. I divide the number into periods of two figures each, beginning at the place of units.

2ndly. I find the greatest square contained in the first or left-hand period, and place its root in the quotient, and subtract the square itself from that period; and, to the remainder I annex the second period for a dividend.

3dly. I place the double of the root already found on the left hand of the dividend for a divisor, by which I divide the dividend, omitting the place of units, and place the result both in the root, and on the right of the divisor; then, by it I multiply the divisor thus completed, and subtract the product from the dividend, and, to the remainder I annex the next period for a new dividend.

4thly. To the completed divisor I add the figure last put in the root; the sum is a new divisor, with which I proceed as before.

Note. *If there be decimals in the given quantity, it must be pointed both ways from the decimal point; and, when the figures belonging to the given number are exhausted, the operation may be continued to any length by annexing ciphers.*

352. What is the square root of 531441?

```
        ˙ ˙ ˙
       531441(729 Ans.
       49
   142)414
        284
  1449)13041
       13041
       =====
```

353. Repeat the rule for extracting the cube root.

I separate the given number into periods of three figures each, by putting a point over every third figure from the place of units.

Then I find the nearest root to the first period, and put it in the quotient; I subtract its cube, and bring down the next period for a new dividend, and to find the remaining figures of the root.

1st. I square the quotient and triple the square for a divisor.

2ndly. I find how often this divisor is contained in the dividend, rejecting the units and tens, which will give the second figure of the root.

3rdly. I square the last figure of the quotient, and put it as the units' and tens' places of the divisor.

4thly. I multiply the triple of the last figure, in the quotient by the others; which product I place under the divisor, putting units under tens, and tens under hundreds, &c. the sum of these two numbers I multiply by the last figure of the quotient, I subtract the product from the dividend, and, to the remainder I bring down the next period for a new dividend, with which I proceed as before.

Note. *The same rule must be observed for continuing the operation and pointing for decimals as in the square root.*

354. What is the cube root of 387420489?

$$\dot{3}87\dot{4}20\dot{4}89(729\ \textit{Ans.}$$
$$343$$
$$7^2 \times 3 = 14704) \quad 44420$$
$$2 \times 3 \times 7 = \quad 42$$
$$15124 \times 2 = 30248$$
$$72^2 \times 3 = 1555281 \quad)14172489$$
$$9 \times 3 \times 72 = \quad 1944$$
$$1574721 \times 9 = 14172489$$

APPENDIX.

Tables.

I. MONEY.*

4 farthings	—	1 penny
12 pence	—	1 shilling
20 shillings	—	1 pound

II. AVOIRDUPOIS WEIGHT.

16 drams	—	1 ounce
16 ounces	—	1 pound
28 pounds	—	1 quarter
4 quarters (112lbs.)	-	1 hundred weight
20 hundred weight	—	1 ton
19½ hundred weight	—	1 fodder of lead

* I have not given an extended table of farthings, pence, and shillings, as is generally done; for, if the pupil can perform division of simple numbers with facility, he can be at no loss whatever to divide farthings by 4, pence by 12, and shillings by 20, mentally; besides, in using a table, he is obliged to be more indebted to his memory than to his judgment.

III. PRACTICE TABLE.

The aliquot parts of a pound sterling.

s. d.			s. d.		
10 0	are the	half	3 4	are the	sixth
6 8	—	third	2 6	—	eighth
5 0	—	fourth	2 0	—	tenth
4 0	—	fifth	1 8	—	twelfth

The aliquot parts of a shilling.

d.			d.		
6	are the	half	2	are the	sixth
4	—	third	$1\frac{1}{2}$	—	eighth
3	—	fourth	1	—	twelfth

The aliquot parts of a ton.

cwt.			cwt.		
10	are the	half	$2\frac{1}{2}$	are the	eighth
5	—	fourth	2	—	tenth
4	—	fifth	1	—	twentieth

The aliquot parts of a cwt.

qrs. lbs.			lbs.		
2 or 56	are the	half	16	are the	seventh
1 or 28	—	fourth	14	—	eighth

The aliquot parts of a quarter.

lbs.			lbs.		
14	are the	half	4	are the	seventh
7	—	fourth	$3\frac{1}{2}$	—	eighth

IV. TROY WEIGHT.

24 grains	—	1 pennyweight
20 pennyweights	—	1 ounce
12 ounces	—	1 pound

V. APOTHECARIES WEIGHT.

20 grains	—	1 scruple
3 scruples	—	1 dram
8 drams	—	1 ounce
12 ounces	—	1 pound

VI. WOOL WEIGHT.

7 pounds	—	1 clove
2 cloves	—	1 stone
2 stones	—	1 todd
$6\frac{1}{2}$ todds	—	1 wey
2 weys	—	1 sack
12 sacks	—	1 last

VII. DUTCH WEIGHT.

16 drops	—	1 ounce
16 ounces	—	1 pound
16 pounds	—	1 stone

VIII. CLOTH MEASURE.

$2\frac{1}{4}$ inches	—	1 nail
4 nails (9 in.)	—	1 quarter

3 quarters (27 in.)	—	1 Flemish ell
4 quarters (36 in.)	—	1 yard
4 quar. $1\frac{1}{5}$ in. ($37\frac{1}{5}$ in.)	—	1 Scotch ell
5 quarters (45 in.)	—	1 English ell
6 quarters (54 in.)	—	1 French ell

IX. LONG OR LINEAL MEASURE.

3 barley corns	—	1 inch
12 inches	—	1 foot
3 feet	—	1 yard
2 yards (6 feet)	—	1 fathom
$5\frac{1}{2}$ yards	—	1 pole
4 poles	—	1 chain
10 chains (40 po.)	—	1 furlong
8 furl. (1760 yds)	—	1 mile
3 miles	—	1 league
20 leag. (geographical)		1 degree
$23\frac{1}{6}$ leag. (Eng. stat)	—	1 degree
60 geographical miles		1 degree
$69\frac{1}{2}$ statute miles	—	1 degree
360 degrees		the circumference of the earth

X. SQUARE MEASURE.

144 square inches	—	1 square foot
9 square feet	—	1 square yard
$30\frac{1}{4}$ square yards	—	1 square pole
40 square poles	—	1 square rood
4 square roods	—	1 square acre
640 square acres	—	1 square mile

100 square feet	—	1 sq. of flooring
272¼ square feet	—	1 rod of brickwork
16 square poles	—	1 chain
10 square chains	—	1 acre
4840 square yards	—	1 acre
30 acres	—	1 yard of land
100 acres	—	1 hide of land
40 hides	—	1 barony

XI. CUBIC OR SOLID MEASURE.

1728 solid inches	—	1 solid foot
27 solid feet	—	1 solid yard
40 solid ft. rough timber		1 ton or load
50 solid ft. hewn timber		1 ton or load
108 solid feet	—	1 stack
128 solid feet	—	1 cord

XII. WINE MEASURE.

4 gills	—	1 pint
2 pints	—	1 quart
4 quarts	—	1 gal. (231 so. in.)
10 gallons	—	1 anker, or keg
18 gallons	—	1 runlet
31½ gallons	—	1 barrel
42 gallons	—	1 tierce
63 gallons	—	1 hogshead
84 gallons	—	1 puncheon
2 hogsheads	—	1 pipe, or butt
2 pipes	—	1 tun

XIII. ALE AND BEER MEASURE.

4 gills	—	1 pint
2 pints	—	1 quart
4 quarts	—	1 gallon
8 gallons	—	1 firkin of ale
9 gallons	—	1 firkin of beer
2 firkins	—	1 kilderkin
2 kilderkins	—	1 barrel
3 kilderkins	—	1 hogshead
3 barrels	—	1 butt
48 gallons	—	1 hogshead of ale
54 gallons	—	1 hogshead of beer

XIV. COAL MEASURE.

4 pecks	—	1 bushel
3 bushels	—	1 sack
12 sacks, or 36 bush	—	1 chaldron
21 chaldrons	—	1 score

XV. DRY MEASURE.

2 pints	—	1 quart
2 quarts	—	1 pottle
2 pottles	—	1 gallon
2 gallons	—	1 peck
4 pecks	—	1 bushel
3 bushels	—	1 sack
8 bushels	—	1 quarter
5 quarters	—	1 wey, or load
2 weys	—	1 last

4 bushels	—	1 coomb
10 coombs	—	1 wey
36 bush. or 12 sacks	—	1 chaldron

XVI. MEASURE OF TIME.

60 seconds	—	1 minute
60 minutes	—	1 hour
24 hours	—	1 day
7 days	—	1 week
4 weeks	—	1 month
13m. 1d. 6h. or 365¼d.	—	1 Julian year
365d. 5h. 48m. 48s.	—	1 Solar year
52 1/7 weeks	—	1 year of 365 days

Thirty days have September,
April, June, and November;
February, twenty-eight alone,
And all the rest have thirty-one;
But, Leap-year, coming once in four,
Gives February one day more.

XVII. MOTION OF THE EARTH, REDUCED TO TIME.

360 degrees equal to	—	24 hours
15 degrees	—	1 hour
1 degree	—	4 minutes

XVIII. APPARENT MOTION OF THE SUN, REDUCED TO TIME.

360 deg. or 12 signs equal to	—	365¼ days, nearly
30 degrees or 1 sign	—	30 days, do.
1 degree	—	1 day, do.

XIX. MOTION.

60 seconds	—	1 minute
60 minutes	—	1 degree
90 degrees	—	1 quadrant
4 quadrants	—	1 circle

XX. PAPER.

24 sheets	—	1 quire
20 quires	—	1 ream
21½ quires	—	1 Printer's ream

XXI. QUARTERLY TERMS,

IN ENGLAND.

Lady-day	March 25th.
Midsummer	June 24th.
Michaelmas	Sept. 29th.
Christmas	Dec. 25th.

IN SCOTLAND.

Candlemas	Feb. 2nd.
Whitsunday	May 15th.
Lammas	Aug. 1st.
Martinmas	Nov. 11th.

XXII. NUMERATION TABLE.

Of Quintillions.						Of Quadrillions,						Of Trillions.						Of Billions.						Of Millions.						Of Units.					
h. of th.	t. of th.	th.	h.	t.	u.	h. of th.	t. of th.	th.	h.	t.	u.	h. of th.	t. of th.	th.	h.	t.	u.	h. of th.	t. of th.	th.	h.	t.	u.	h. of th.	t. of th.	th.	h.	t.	u.	h. of th.	t. of th.	th.	h.	t.	u.
1	2	3,	4	5	6.	7	8	9,	8	7	6.	5	4	3,	2	1	2.	3	4	5,	6	7	8.	9	8	7,	6	5	4.	3	2	1,	2	3	4.

The above number is read as follows: one hundred and twenty-three thousand, four hundred and fifty-six *Quintillions;* seven hundred and eighty-nine thousand, eight hundred and seventy-six *Quadrillions;* five hundred and forty-three thousand, two hundred and twelve *Trillions;* three hundred and forty-five thousand, six hundred and seventy-eight *Billions;* nine hundred and eighty-seven thousand, six hundred and fifty-four *Millions;* three hundred and twenty-one thousand, two hundred and thirty-four.

XXIII. MULTIPLICATION TABLE.*

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12 *a*	14	16 *b*	18 *c*	20 *d*	22	24 *e*
3	6	9	12 *a*	15	18 *c*	21	24 *e*	27	30 *f*	33	36 *g*
4	8	12 *a*	16 *b*	20 *d*	24 *e*	28	32	36 *g*	40 *h*	44	48 *i*
5	10	15	20 *d*	25	30 *f*	35	40 *h*	45	50	55	60 *j*
6	12	18 *c*	24 *e*	30 *f*	36 *g*	42	48 *i*	54	60 *j*	66	72 *k*
7	14 *a*	21	28	35	42	49	56	63	70	77	84
8	16	24 *e*	32	40 *h*	48 *i*	56	64	72 *k*	80	88	96
9	18 *c*	27	36 *g*	45	54	63	72 *k*	81	90	99	108
10	20 *d*	30 *f*	40 *h*	50	60 *j*	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24 *e*	36 *g*	48 *i*	60 *j*	72 *k*	84	96	108	120	132	144

* The inventor of this table is said to be Pythagoras of Samos, hence it is frequently stiled the Pythagoric Abacus.

a 6 times 2 = 12; and 4 times 3 = 12 also
b 8 times 2 = 16; and 4 times 4 = 16.
c 9 times 2 = 18; and 6 times 3 = 18.
d 10 times 2 = 20; and 5 times 4 = 20.
e 12 times 2 = 24; and 8 times 3 = 24; and, 6 times 4 = 24.
f 10 times 3 = 30; and 6 times 5 = 30.
g 12 times 3 = 36; and 9 times 4 = 36; and, 6 times 6 = 36.
h 10 times 4 = 40; and 8 times 5 = 40.
i 12 times 4 = 48; and 8 times 6 = 48.
j 12 times 5 = 60; and 10 times 6 = 60.
k 12 times 6 = 72; and 9 times 8 = 72.

It may be of some use to the pupil, while learning the above table, to observe that the product of all the odd numbers in 5 times ends with 5; and that of the even numbers with a cipher; or 5 times any number is just half of 10 times the same: the product of any number by 10 is just that number with a cipher annexed; and the product of any of the nine digits by 11 is just two of them, the one considered as the units' place of the product, and the other as the tens'.

Although I do not approve of difficult calculations being put into the hands of beginners, yet, as it is not unlikely that this small work may meet the eye of those that are *advanced* in Arithmetic, I have, therefore, given a few amusing questions for exercise in leisure hours, which are as follows:—

I.

On Christmas week a Bond-street blade
Ran through one-third his income;
Besides ten pounds he gave to Ned—
Ned bowed, and said 'twas handsome.
With heavy heart, this spendthrift elf
Went home to count his cash, sir,
He found but half his income left,
With fifteen pounds to dash sir.
What was his income? *Ans.* £150.

2.

As I was beating on the forest grounds,
Up starts a hare before my two greyhounds;
The dogs, being light of foot, did quickly run
Unto *her* fifteen yards just twenty-one,
The distance that she started up before
Was four-score sixteen yards just, and no more;
Now, I would have you unto me declare
How far they ran before they caught the hare?
Ans. the dogs ran 336 yards and the hare 240 yards.

3.

A draper sold cloth at 11s. per yard, by which he cleared $\frac{3}{8}$ of his money; he afterwards raised it to 13s. 9d. per yard; what did he clear per cent by the latter price?

Ans. 100 per cent.

4.

The Race-horse, Thunderer, and Nonsuch, men of war were engaged to demolish a fort which lay on the frontiers of the enemy, for which they were to receive £14000 to be divided amongst them in the proportion of $\frac{1}{2}$, $\frac{1}{4}$, & $\frac{1}{8}$ respectively; but, Nonsuch having foundered at sea, the other two gallantly performed the task. It is required to divide the whole sum properly between the other two?

Ans. The Race-horse, £9333 6s. 8d. and Thunderer, £4666 13s. 4d.

5.

Divide £176 between A and B. and give A 5 guineas and a half more than B*

Ans. A £90 17s. 9d. and B £85 2s. 3d.

6.

Miss Forbes can finish a piece of needle-work in 6 days, (working 7 hours a day) and Miss Jones takes 8 days to do the same; in what time will they both do it working together?

Ans. 3 da. 8 ho.

---

* Gordon, in his 'Institutes of Arithmetic' has given a similar question to this in Single Position; and Wiseman, in his 'Arithmetician's Text Book', gives a like question in Double Position. It is certainly much easier to take the difference of the two shares from the amount, and divide this by 2 for the less share, and add the difference to the less share for that of the greater. Or, half the sum of the amount and difference will give the share of the greater, and, the difference taken from this will leave the share of the less. Or, from half the amount take half the difference for the less share, and, to this add the difference for the greater share.

7

Supposing Miss Campbell could count a certain sum of money in 3 days, Miss Nesham in 4 days, and Miss Glegg in 6 days, telling 9 hours a day; in what time will the three young ladies be able to finish this task if they all do it together? *Ans.* 1 da. 3 ho.

8

What capital will produce £20 in 2 years at $3\frac{3}{4}$ per cent?

Ans. £266 13s. 4d.

9.

Four Persons met at the foot of Benlomond, and purposed reaching its summit before sunrise, that they might have the pleasure of seeing Sol mount with majestic grandeur.—A had 5 pints of Highland whiskey, B $4\frac{1}{2}$, C $3\frac{1}{2}$, and D who was an expert accountant told the others that he had no whiskey with him, but, that if they allowed him to partake equally with themselves, that his exact proportion would be 10s. 10d. this they agreed to, but quarrelled about the division of the money, it was therefore referred to the decision of D; required the share of each?

Ans. A got 5s. 10d. B 4s. 2d. and C. 10d.

10.

Suppose one-third of six be three,
What should one-fourth of twenty be?

Ans. $7\frac{1}{2}$.

11.

A house was bought by A, B, C,
Who paid for it full dear,

They gave two thousand pounds in all,
 In manner stated here;
Two-thirds of B's share A laid out,
 And B three-fifths of C's,
How much did each pay of the same;
 Pray solve it, if you please?

Ans. A paid £400, B £600, and C £1000.

12.

In 1759 Jennison Shafto, Esq. rode a match against time, at Newmarket, $50\frac{1}{4}$ miles in 1 hour, 49 minutes, 17 seconds; what ground did he pass over in a second?

Ans. 13 yds. 1 ft. $5\frac{1}{2}$ in.

13.

A brazen lion being placed in an artificial fountain, conveys water into a cistern by a stream issuing from his mouth, by two from his eyes, and by another from the bottom of the right foot, the pipes through which these streams pass are of different capacities, in such sort that by the foot set open alone, the cistern can be filled in 12 hours, by the mouth alone in 2 hours, by the right eye in 3 hours, and by the left eye in 4 hours; in what time can the cistern be filled if all these streams be set open at once?

Ans. 51 min. 25 sec.

14.

A gentleman hires a servant and agrees to give him £24 and a livery coat for a year's service, at the end of 8 months the servant obtains leave to quit his situation, and for his services receives £13 and his livery coat, which were his full wages for that time; what was the coat valued at?

Ans. £9.

15.

A lion, a bear, and a wolf meeting with a sheep that had strayed from the flock, altogether fell on it; now, supposing the lion alone could eat it in 4 hours, the bear in 6 hours, and the wolf in 8 hours; how long would they all be in devouring the sheep?

Ans. 1 ho. 50 min. 46 sec.

16.

I bought an ox, a cow, and calf
For twenty sovereigns and a half,
The cow in value, (let me see),
Was to the ox as one to three;
The price that for the calf was given,
Was to the cow as two to seven;
The question, then, I ask of you,
What cost they each, calf, ox, and cow?

Ans. calf £1 7s. 4d. ox £14 7s. and cow £4 15s. 8d.

17.

What is the difference between the interest and discount of £50,000 (the yearly sum offered to the Queen) for 7 years, at 5 per cent. per annum?

Ans. £4537 0s. 9d.+

18.

What is the difference between the compound and simple interests of £400 for 50 years, at 5 per cent. per annum?

Ans. £3189 12s. 4$\frac{3}{4}$d.

19.

It is now between 5 and 6 o'clock, and the hour and minute hands are together; what is the hour of the day?

Ans. $27\frac{3}{11}$ min. past 5 o'clock.

20.

The hour and minute hands of a watch are exactly in conjunction at twelve o'clock; when do they next come in contact?

Ans. $1\frac{1}{11}$ ho. or 5 m. $27\frac{3}{11}$ sec. past 1 o'clock.

21.

Two persons, A and B. being on opposite sides of the circle in Regent's Park, (which suppose to be a mile in circuit) they begin to go round it both the same way, at the same instant of time: A at the rate of 100 yards per minute, and B. 600 yards in 5 minutes; how many times will they have traversed the circle before the quicker overtake the slower?

Ans. 3 times.

22.

Standing on the side of a river, I found that a line stretched from the top of a precipice rising perpendicularly 449 feet, on the other side measured 585 feet; tell me the breadth of the river.

Ans. 375 feet.

23.

If 12 men can build a wall 60 feet long, 4 feet thick, and 20 feet in height, in 24 days, working 12 hours per day; what length of wall, 3 feet thick, and 12 feet high, can 18 men build in 18 days, working 8 hours per day?

Ans. 100 feet.

24.

The amount of a certain sum of money laid out to interest is £6000; the principal is just 5 times the interest; what is the principal and interest, and how long has it been at interest, allowing 5 per cent?

Ans. prin. £5000; int. £1000; and 4 yrs.

25.

In what time would a stone, falling from the top of a rock, that stands perpendicularly above the sea $144\frac{3}{4}$ feet, reach the surface of the water?

Ans. 3 seconds.

Note. *It has been found by exact experiment, that, a body, in the latitude of London, falls* $16\frac{1}{12}$ *feet in the first second of time; and, as* $16\frac{1}{12}$ *feet* : 1^2 *second* :: *the given space to the square of the seconds required.*

26.

What space will a heavy body falling freely, pass through in 12 seconds?

Ans. 2316 feet.

27.

A brigade of cavalry consisting of 648 men, is to be formed into a long square, having 24 in front; how many ranks will there be?

Ans. 27 ranks.

28.

Covetous Sam was detected stealing apples by mad Tom, to appease Tom, he gave him half the number he had stolen, but Tom was generous enough to return him 10; in his way home, he was attacked by raving Ned, who robbed him of half what he had left, Ned very condescendingly returned him 4; afterwards, unluckily, positive Jack met him, and demanded half of what he had then in his possession, Jack very politely returned him 1, then he found he had but 18 left;—how many apples did this very unlucky depredator steal?

Ans. 100.

29.

If I were on the north side of the river Clyde, directly opposite to the Greenock battery, what time will elapse between seeing the flash, and hearing the report of one of the cannons, the distance being 3 miles, 7 furlongs, 5 poles, $4\frac{1}{2}$ yards?

Ans. 18 seconds.

Note. *When sound is not interupted, it is found by experiment to move uniformly* 1142 *feet in* 1 *second of time.**

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* Hutton in his course of mathematics, observes, that the interval between seeing the flash of a gun or lightning and hearing the report, may be ascertained by counting the pulsations of the wrist, allowing 70 to a minute, in persons of moderate health; or, $5\frac{1}{2}$ pulsations to a mile.

30.

Standing in Kensington Square, I observed, that, between seeing the flash of lightning and hearing the thunder, $\frac{3}{4}$ of a minute had elapsed; at what distance from the square was the thunder? *Ans.* 9 mi. 5 fur. 34 po. 3 yds.

31.

If 12 oxen eat up the grass of $3\frac{1}{3}$ acres in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, how many oxen will eat 24 acres in 18 weeks, the grass being allowed to grow uniformly?

Ans. 36 oxen.

32.

If 12 oxen eat up the grass of $3\frac{1}{3}$ acres in 4 weeks, and 21 oxen eat 10 acres in 9 weeks, how many oxen will 24 acres maintain for ever, the grass being supposed to grow uniformly?

Ans. $21\frac{3}{5}$ oxen.

33.

What is the text in the Old Testament, where the verse is three-fourths of the book, the chapter as much as the book, and seven-fifteenths of the verse, and the sum of the book, chapter, and verse is 62?

Ans. *A continual dropping in a rainy day, and a contentious woman are alike.*

34.

Tell me the text in the New Testament, when the chapter is twice the book, the verse is the sum of the book and chapter, and the sum of the book, chapter, and verse, is 36?

Ans. *If it be possible, as much as lieth in you, live peaceably with all men.*

INDEX.

Errata.

Page 33, *line* 3, *for* denominations, *read* denominators.

Page 34, *line* 10, *for* $32\frac{6}{16}$ *read* $\frac{6}{16}$

Page 37, *line* 23, *for* ule *read* rule.

Page 54, *line* 10, *for* terms *read* third terms.

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