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Partnership Mathematics Content Courses for Prospective and Practicing Elementary and Middle School Teachers

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Abstract

Since fall 1995 the Department of Mathematical Sciences at New Mexico State University has been involved in a partnership with the Las Cruces, NM, USA, Public Schools. We offer a series of five one-semester courses attended jointly by prospective and practicing teachers of grades K- 8. The sequence covers the arithmetic of integers, rational, and real numbers, metric geometry, algebra, and science, with integrated use of technology. Practicing teachers act as mentors for prospective teachers, who become their apprentices. Material is organized into units that provide lesson plans for elementary and middle grades. Practicing teachers use them in their classrooms, and their apprentices observe or even teach under their mentors' supervision. The partnership supports preservice teachers as they learn advanced mathematics and then use it in a classroom setting.

Background for the project

There is no central agency that controls schools in the United States. Individual states, school districts, and even schools have considerable freedom in setting their own programs and curricula. Teacher licensing is done by individual states, and requirements vary. Teacher preparation is provided by Schools of Education, which are part of the college system. These Schools concentrate on pedagogical and educational issues. Departments located in Colleges of Arts and Sciences offer courses in subject matter, such as mathematics or the sciences.

But elementary teachers are generalists who must teach all subjects, even if during their college years they do not have sufficient time to acquire enough subject matter knowledge (Stigler & Hiebert, 1999). Also the format of university courses is very different from the format of classroom work in the early and middle grades (Lappan, 2000). This provides a formidable obstacle for young teachers.

Our experience with workshops and summer courses for teachers convinces us that such opportunities do not provide enough learning time to be the basis for sustained continuation of professional development. The idea of a school district-university partnership is not new (see e.g. Sirotnik & Goodlad, 1988). The partnership program described here seems to be a solution to the problem of too little time for professional development. It provides continuing education for practicing teachers, and offers future teachers courses in mathematics that cover topics they will need to know in their classrooms in a setting similar to the settings found in schools. The most encouraging aspect is the response of practicing teachers who are willing to spend two evenings per week in order to continue their professional education and improve their qualifications.

Organization of the university courses

The courses are taught in a laboratory format. This means that there are no lectures longer than five to ten minutes. Students sit at tables in groups of four to six, and during the class session they do the tasks assigned to them. The only whole-class activities are discussions and classroom reports (brief presentations by students and teachers). In the smaller partnership classes (12-30 students), in addition to the instructor, a teaching assistant is always present; and the larger classes (30-50) need two teaching assistants. The teaching assistants are either graduate students specializing in mathematics or education, or teachers who have already taken a number of the partnership courses.

The mentor-apprentice partnership

University undergraduate students and practicing teachers team up together in an apprentice-mentor relationship. During a semester an undergraduate student usually has two or more teacher mentors, who teach two different grade levels. Each mentor has up to four apprentices at a given time. When there are not enough mentors in the class, teachers who have previously taken the class and who are not currently enrolled at the university serve as mentors. Thus the relationship between the teachers and the university is not limited just to the semesters when teachers are actually enrolled in the university courses.

Obligations of teachers and students

Both students and teachers are obliged to attend the university class, and they work together on activities. Seating is arranged so that at least one teacher is in each group. If there are not enough teachers, the teaching assistants sit with some students.

The advantages to writing about mathematics are well documented (e.g., Connolly & Vilardi, 1989; Morgan, 1998). Both students and teachers keep journals, writing about the mathematical topics in the activities covered in each session. `Journals are collected about every three weeks; the instructor reads them and provides individual feed back.

Examples of journal entries and instructor comments

(1) Exploration of stars (This journal entry was written by a preservice teacher.)

My understanding of the unit. We had to divide 360 degrees by the number of points the star would have, so we would have the measure of the angle. The first star we made had five points. We divided 360 degrees by 5 to find the distance apart of the tick marks, namely, 72 degrees. For the star with six points 360 degrees was divided by 6, for an angle of 60 degrees. 360 degrees divided by 7 equals about 51.4, which we rounded to 51.5 degrees for the seven pointed star. After we measured the angles using a protractor, we had to join them up. We first numbered the points, then we joined them together. For the 5-pointed star, the first point connected to points 3 and 5. The third point also connected to the 5th point which connected to the 2nd point which connected to the 4th point, creating the star. Mathematics. Division, angles, degrees, protractors, rounding, geometry, radius, even and odd.

My reaction. I really enjoyed this unit. I had a little trouble at first with the 5 star. I could not get the angles right, but I eventually measured them correctly. I also continued the pattern inside each of the stars, making smaller and smaller stars.

Instructor's comments. Nice writing! Do you know how many different 7-point stars there are? You can make two different ones, a fat one and a skinny one; it depends on which points you join. Also did you notice that the five-point star does not "split" into two pieces, but the six-point star does "split"?

(2) Understanding long division (Again, this was written by a preservice teacher.)

To logically understand the simplicity of long division, we picked apart the written algorithm. Let's divide 17682 by 246. First we used a calculator to find the multiples of 246. Making a list of multiples makes it more visual to see how it works which is a very effective step. As listed below:

1	*246*	*4*	*984*	*7*	*1722*
2	*492*	*5*	*1230*	*8*	*1968*
3	*738*	*6*	*1476*	*9*	*2214*

Second we set up the problem as follows below, which is usually the step that is left out of solving long division:

17682		
-17220	*(70)*	
462		
-246	*(1)*	*Instructor comment:*
216.0		*Now add up the*
-196.8	*(.8)*	*numbers in*

19.20 *parentheses to get a*

-17.22 *(.07)* *quotient.*

This is how a traditional long division problem would be set up:

```
      71.87
246)17682.
   -17220
     462
    -246
     216.0
    -196.8
      19.20
     -17.22
```

Instructor comment: Traditionally the 0 on 17220 and the decimals (except those in the dividend and quotient) would not be included. Very nice work!

By breaking down the algorithm in this way, the student shows the correctness of the procedure that was employed. (For understanding algorithms, see also Ma, 1999.)

Students are given both obligatory and optional homework, and are required to make at least ten visits to the classrooms of their mentors. During their first visit they usually observe; later they co-teach; and finally they teach under the supervision of their mentors. But the decision about the role that a student will have in a teacher's classroom is the responsibility of the teacher, and not the university instructor. All students are required to describe each classroom visit in their journals.

Teachers do not evaluate students' performance. Evaluations and assignment of grades are done only by the instructor, who also gets input from the teaching assistants.

Example of a homework assignment

Each student selected an irregularly shaped block of wood, and the task was to find the surface area and volume of the block, to make at least three different drawings of the block, showing measurements, and to explain each step used in finding the answer. We decided that some blocks were more difficult to measure than others, so the instructor assigned a difficulty level (one = easiest; three = hardest) to each block.

This homework, by a preservice teacher, received a score of 100%, with a difficulty level of two. It included three more pages of drawings and calculations:

In order to find out the area and the volume of my chunk of wood, I did the following. I measured the length and widths for sides A, B, C, and D. For side E, I cut it to make a rectangle and a triangle. (See attached papers.) I found the length and widths for these.

*I then found out all the areas for these sides (see paper). I then added up all the areas & got the total area for the block (40.88 sq. in.). In order to find the volume I formed the volume of the rectangle [instructor: rectangular solid], using the formula V = L*W*H. I then found the volume of the triangle [instructor: triangular prism] by making it an imaginary rectangle [instructor: rectangular solid]. I found the volume for the [rectangular solid] and then divided by two to find the volume of the [triangular prism]. I then added the volume of the [rectangular solid] and the [triangular prism] and got the total volume of my chunk of wood (15.75 cubic inches). (See attached paper for all calculations and measurements.)*

Teachers mentor their university student apprentices in classrooms with their pupils. During the students' visits they typically use materials that they have gone over together in the university courses, so students see how experienced teachers use the material that they have just studied. Often the lessons from the university courses have never been tried with pupils before. When teachers try new lessons, they administer diagnostic tests to pupils to assess their understanding of the mathematical concepts.

Rationale for this format for the courses

We consider that teachers' education is professional education. Teachers will use the subject knowledge learned in college over and over again. So they must learn in a very detailed way the material they are going to teach. "General ideas" are not good enough. This suggests that future teachers' mathematics courses should be in a laboratory format in which students work under the supervision of an instructor or an experienced teacher. In such courses the amount of material covered is smaller than in a typical College of Arts and Sciences undergraduate course, but it is covered in more detail and with a larger emphasis on skills. When these students become teachers, they will use their subject matter knowledge in their classrooms. Seeing how their mentors do it, and teaching under their supervision, provides them with the practice needed in any professional education. Classroom-ready lesson plans are important for both teachers and students. For teachers they provide something that is immediately usable, and for students they connect what they learn in the university setting to what they see in their mentors' classrooms.

Logistics

The courses are offered during the school year, when public schools are in session, so units can be immediately tried in schools. Courses are held late in the afternoon or evening so teachers can attend. The Las Cruces Teachers' Center distributes course announcements to schools in spring and fall, helping to recruit teachers for the classes. Teachers' university tuition is funded by grants, which have also covered the cost of tools, calculators, other supplies, and Xeroxing of class handouts.

Starting a partnership program requires the approval of three groups: (1) The School District must allow undergraduates to visit classrooms, and it must allow teachers to use the lesson plans we provide. It also must give us access to the work of pupils in schools so we can assess what they learn. (2) The University's College of Education must approve the courses as appropriate for both present and future teachers. (3) The University's Department of Mathematical Sciences must approve the content of the courses, and it must agree that the same course can be attended simultaneously by graduate students (teachers) and undergraduates (future teachers).

The content of the courses was chosen in close collaboration with the School District. The District identified middle school algebra and the use of technology as two critical issues that needed to be included in any attempt to improve the mathematical education of its pupils. The integration of mathematics and science education, which is the topic of the newest course in the series, is high on the District's priority list.

Course material

The material is organized into units. Most units are based on a specific task that cannot be completed without applying mathematical knowledge and skill.

Example. (A class project.) From poster board construct 20 cubes having volumes 1, 2, 3, …, 20 cubic inches. In order to do the task students must learn the formula for the volume of a cube. They have to compute cube roots. And they have to have enough geometric skills and knowledge to draw the plans and assemble the cubes.

Each unit is accompanied by a lesson plan for use in school classrooms.

Students and teachers are given about 50 units each semester (see Baggett & Ehrenfeucht, 1995; 1998; 2001; in press). Students and teachers go through them either in the university class or at home. Teachers use lessons of their choice from the university class in their own classrooms. A typical unit requires one or two periods of school time. There is no textbook that covers the mathematical topics in the abstract; all topics are learned in the context of some application.

Mathematical content

The sequence of courses covers the following mathematical topics: (1) Arithmetic of real numbers and its subsets, rational numbers, integers and whole numbers. (2) Three dimensional metric geometry, namely, geometry based on the concept of distance. This is a modern version of the "practical geometry" of the past. (4) Algebra in the Newtonian (and not Eulerian) tradition. It doesn't stress the "formal aspects" of algebra, but relates it to numerical techniques and physical quantities. (5) Recording and analyzing measurements in the physical sciences (e.g., mass, energy, force, speed, acceleration, and so on), with some trigonometry. All topics are presented in a mutually consistent way. Here is an example: Unacceptable: 3 is the next number after 2 (it is unacceptable because $2 < 2.5 < 3$). Acceptable: 3 is the next integer after 2.

All topics are present in all five courses. But the first course focuses on arithmetic, the second on geometry, the third on algebra, the fourth on the use of technology, and the fifth on science (mainly physics).

The use of four-operation calculators is integrated even with mathematical topics that are usually taught in kindergarten and first grade (Baggett & Ehrenfeucht, 1992). Scientific calculators are used with materials containing algebra, and graphing calculators are used for computationally complex tasks and "computer simulations". Computers are used for mathematical tasks, internet searches, and so on. The technology used in the five courses differs. The first two courses use four-operation calculators, the third adds scientific calculators, and the fourth and fifth add graphing calculators and a computer lab.

Early in the courses students design algorithms and learn rudiments of programming. Already in the first course they learn how to compute a cube root on a four-operation calculator, using an iterative procedure based on the fact that (the cube root of n) = lim Z(k), where Z(k+1) = (the fourth root of n*Z(k)). In the third course, focussing on algebra, they are shown a procedure to solve equations using Newton's method. Without modern technology, introducing mathematical topics and sequencing them are limited by the slow pace that children acquire minimal skills in written computation. Technology permits topics to be selected on the basis of children's intellectual development and interests, rather than on the level of their computational skills.

The role of measurements and use of tools

One way to build number sense in the early grades is to show pupils that numbers come from measurements, which has always been done in learning mathematics as a vocation (Daboll, 1812; Pike, 1827). Thus measurements with common standard measuring tools, rulers, measuring tapes, protractors, scales, measuring cups, and thermometers are important parts of lessons for all grades. Their proper and skillful use is considered important. Tools used in constructions in metric geometry are not limited to compass and straight edge, but include rulers, protractors, and even French curves.

Discussion

The purpose of mathematics in grades kindergarten to eight is twofold: (1) to provide a solid knowledge of "everyday" mathematics, and (2) to provide a background for future study for pupils who will later choose "mathematically intensive" careers.

We think that these two different goals can be achieved best if mathematics is taught in a unified and consistent way as an applied science. So we do not "explain" mathematics in terms of the manipulation of physical objects; instead we present it as "problem solving tool" for a variety of practical problems (Freudenthal, 1973; Nunes, Schliemann & Carraher, 1993). This approach serves both students who like mathematics, and those who struggle with abstract concepts and become "math phobic".

Our approach to technology is also pragmatic. Adults use technology extensively in tasks requiring mathematics; therefore it should be integrated into school learning. We treat technology as a problem solving tool, and not as a "teaching aid." Four-operation calculators are easy to use, and are the most useful in combination with mental calculations. When pupils start to rely on written formulas and learn some algebra, scientific calculators are the most useful. After this, pupils start to explore the variety and versatility of more advanced technology using graphing calculators or computers.

The courses use extensive handouts. Recently we gave students an (anonymous) questionnaire asking if a textbook would be useful for these courses. The large majority answered no. The reason they gave most often was that textbooks in college courses mainly help the instructors, but not the students.

Use of the materials in classrooms

Teachers use the lesson plans provided in the university courses in their K-8 classrooms. But different grades require material with different mathematical content and difficulty. Teachers resolve this problem in two ways. First, they decide which material is appropriate for their pupils. Second, they adapt the material to their grade level.

Example.

Basic task. Students are given a baseball that just fits in a cubic box, a package of rice, and scales. Question: What percentage of the volume of the box is filled by the ball? (The surprising answer is pi/6, or about 52%.)

Method. Weight the ball in the box, fill the empty space with rice, and weigh again. Weigh the empty box, and the box filled with rice. Compute the answer using a simple calculator. This lesson is suitable for grades 5-8. For earlier grades, the problem may be formulated in terms of part of the volume rather than percentage, and students may be given measuring cups, to avoid the difficulty of computing the ratio of volumes from weights. In grades 7 and 8, it can be an introduction to the formula for the volume of a sphere, or even to computing volumes by Cavalieri's principle, using the definite integral on graphing calculators.

Remark. We have found that even very experienced teachers can rarely prepare lesson plans from scratch on a daily basis. But they are usually skilled in adapting existing lessons to the level of their pupils.

Testing and evaluation

Teachers' and students' journal writings describing units studied in the university class are evaluated according to the following criteria: (1) mathematical correctness; (2) completeness of description (lack of omissions); (3) organization; (4) correct grammatical and stylistic embedding of mathematical formulas in the rest of the text; and (5) appropriate drawings and illustrations. (See also Pugalee, 2001.)

We assess in several different ways both the quality of the materials and the skills and knowledge pupils gain from particular lessons that were presented in teachers' classrooms. The university students and teachers give detailed written or oral reports about how a lesson was taught to pupils, what its strong and weak points were, and how it could be improved. When needed, teachers bring in pupils' work, which we can analyze before it is returned. We use follow-up assessment tasks that are non-intrusive, either activities with diagnostic value that test pupils' skills or knowledge, or writing assignments that assess their understanding and recall. For example, a hands-on assignment may be assessed by means of a word problem with similar content. The assessment tasks are administered by the pupils' classroom teachers, and they have the format of a typical lesson. Recall tasks are often given with a delay of several weeks. We find pupils' recalls to be the most useful data for evaluating both the materials and pupils' understanding of it. Finally, both teachers and university students fill out course evaluations which include their subjective evaluations of the usefulness of the materials. Many units are revised several times before a satisfactory version is found because of flaws that are discovered during their testing.

Examples of children's recalls

First graders did two tasks: (1) They drew triangles by making three dots and connecting them with straight segments using a ruler. Later they colored the patterns they made. (2) They were given a non-convex 9-gon and were asked to measure its sides in inches and sixteenths of inches and write the lengths next to each side. Their spoken recalls were taken two days later. Here are two. *Subject 1. I learned to make lines with a ruler, to connect dots and measuring and about fractions. I liked the coloring best. Subject 2. I learned you can make anything out of triangles. And how to measure lines. I liked doing the triangles.* (See Baggett & Ehrenfeucht, 1999.)

Exploration of stars was taught in a sixth grade class, and recalls were requested two days later. Here is a recall of a sixth grader: *Stars: Tools used--calculator compass ruler protractor. Directions- First you need to get all of your tools. Then you set your compass at 4 inches. It needs to be 4 inches because the paper's width is 8 inches. Now you make your circle. Now you have to pick how many points you want for your star. For example you picked a 5 point star. You have to measure where the points are going to be by dividing 360 by 5. You get 72. Now you use your protractor by measuring where 72 is. When you finish doing that you can start making your star. For the last step draw lines from 1 to 3, 3 to 5, 5 to 2, 2 to 4, and 4 to 1. Now you have a star. I thought this was fun. At home I even tried making a harder one. I hope we do this in class again.*

Children remember good lessons well, and procedures are usually remembered better than facts and conclusions (Engelkamp, 1998). A lack of understanding is demonstrated by incoherence in the description of what was done and poor recall.

In this school district, different schools use different textbooks and adhere to different teaching philosophies. Good lessons can be incorporated in a variety of curricula, and fit different teaching styles.

Examples of teachers' and students' opinions from course evaluations

(These comments came from anonymous course evaluations from fall 1999.)

It challenged me to understand more about math and not just math on paper.

Handouts were extremely useful.

I will use the activities for the duration of my career.

This class will definitely help me in the future.

Great content! I can actually use it in my own teaching.

The activities could be used immediately with no modifications.

I was glad to take a class with no textbook.

The course built my confidence in my math ability.

I really learned so much about going into the classrooms and in the different ways teachers teach and children learn.

It was great getting to work outside of class with other teachers.

Conclusions

Teachers taking the courses have directly and successfully used materials from the university classes in their classrooms. From earlier programs we have been involved in, we know that giving teachers only written materials, or discussing them in a few workshops, is not sufficient for a program to succeed. Having a university course, and having both prospective and practicing teachers attend it and go through materials they will use with children, is the key.

Impact on the School District and on other university courses for teachers

The Las Cruces School District enrolls approximately 16000 elementary and middle school students in 29 schools. It employs about 692 elementary and 333 middle school teachers. The elementary teachers teach all subjects. Middle school teachers are at least partially specialists. The District is multi-ethnic and bilingual (Spanish and English).

Since 1995, the first two courses of the series have been offered 12 times, and the last three have been offered 7 times. During this time 159 different teachers (117 of them elementary) have taken the courses, with an average enrollment of over 13 teachers per course. Teachers do not have to take the lower level courses before taking the more advanced ones, so many just take the advanced ones. Here are the data for six years:

Number of courses taken:	1	2	3	4	5
Number of teachers:	105	33	8	10	3

The number of teachers in a school who have taken at least one course varies from 0 to 8.

Ending teachers' professional isolation

Finishing a university course does not end teachers' professional development. Contact with the university is sustained in three different ways:

- Some teachers volunteer to mentor students who take the same class in later semesters.
- The School District holds a yearly summer workshop and "reunion" for all participants.
- Teachers take an active part in preparing and running a yearly Mathematics Education Institute for college mathematics instructors of courses for teachers, organized by the Department of Mathematical Sciences and the College of Education. The fourth annual Institute was held in March 2001. In the four Institutes, about 90 university instructors from New Mexico, other parts of the United States, Central America, and Canada have come to Las Cruces to see the partnership program in action. Teachers allow Institute attendees to visit their classes and see units from the courses presented to their pupils.

The impact on the School District is already visible and significant. About 16% of the elementary teachers have taken at least one university course from the series, and they keep contact with each other and with the university. Many plan to take additional university courses. So this program seems to bring sustained professional development.

In contrast, the impact on other mathematics courses for teachers offered at the college level is very small. Instructors who have a heavy teaching load teach most of the courses using a traditional textbook. For regular faculty the courses are not attractive because they carry little prestige, and they are difficult to teach because very few mathematicians have any experience in elementary education. Finally, on the national level there is little consensus about mathematics in schools. Opinions about the content and methods of school mathematics are not in agreement.

Plans for the future and general conclusions

We already know that average pupils in elementary schools can understand and master mathematics that is more advanced than is usually taught to them, providing that their teachers have a good knowledge of these topics. (See also Butterworth, 1999; Dehaene, 1997; Wynn, 1995; Gelman & Gallistel, 1986.) But we still do not have sufficient data from kindergarten classes, and from middle school algebra classes.

Our current estimate is that elementary teachers need at least four semesters (and preferably more) of college mathematics courses to be adequately prepared to teach this subject. And what mathematics courses they take is of crucial importance. We think courses that cover material related to topics taught in schools should have priority. We also think this can best be achieved if teachers' subject matter education doesn't end in college, but if they continue to take university courses throughout their careers as teachers.

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