Teaching the meaning of arithmetic in early nineteenth-century England, according to teacher William Russell

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Plan for the talk

1. What we know about the author, William S. Russell
   His publications

2. What we don't know about him
   Just about everything else

3. His book *A Companion to Every Treatise on Arithmetic* (1821)
   Its contents
   Its pedagogy
   His concept of the meaning of arithmetic operations

4. Final remarks

1. What we know about William S. Russell

His publications:

A Companion to Every Treatise on Arithmetic (1821)

Superior Small-hand Copies and an Alphabetical Projection (1822)

Appendix to John Walker's *A New System of Arithmetic* (1823)

The Philosophy of Arithmetic, or, A Complete Analysis of Integers (1826)

And apparently
Key to John Walker's *A New System of Arithmetic*
   (we couldn't find it)

A
Companion
TO EVERY TREATISE
ON
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IN THE FORM OF QUESTION AND ANSWER,

IN WHICH

The Science is demonstrated in a clear and familiar manner, with the view of assisting the Teacher, and rapidly advancing the progress of the Pupil:

TO WHICH IS ADDED

An Appendix, containing useful Tables, and numerous Questions for exercise in leisure hours.

BY

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PRIVATE TEACHER OF

Writing, Arithmetic, and Merchants' Accounts,

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St. Mary-le-bone.

The true and only method of promoting Science, is to communicate it with clearness and precision.

Gregory.

London:

Printed for G. & W. B. WHITTAKER, *Ave Maria Lane;* N. HAILES, *Museum, Piccadilly;* G. COWIE & Co. *Poultry;* and P. REYNOLDS, 37, *Wigmore Street, Cavendish Square.*

1821.

Our copy of the book is dedicated in Mr. Russell's calligraphic hand writing to Miss Kinnaird.

SUPERIOR
Small-hand Copies and an Alphabetical Projection.

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DIRECTIONS FOR THE POSITION OF THE BODY, AND FOR HOLDING THE HAND AND PEN:

ALSO,

RULES FOR MAKING EVERY VARIETY OF PEN, AND OBSERVATIONS ON THE CAPITAL AND SMALL LETTERS.

BY WILLIAM RUSSELL,

Author of "A Companion to every Treatise on Arithmetic," and Private Teacher of Writing and Arithmetic.
28, Northumberland Street, New Road, Mary-le-bone.

LONDON:

PRINTED FOR J. HATCHARD, PICCADILLY; SHERWOOD, NEELY, AND JONES, PATERNOSTER ROW;
COWIE AND CO. POULTRY; AND J. SOUTER, ST. PAUL'S CHURCH YARD.

Price)   (One Shilling.

1822.

Small-hand: for young children
Alphabetical projection: handwriting with serifs

Mr. Russell's signature in this book

A

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OF

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Containing

MANY HUNDRED EXAMPLES,

Constructed upon a Plan

Entirely Original.

EXPRESSLY DESIGNED FOR THE ASSISTANCE OF
TEACHERS;

And calculated to facilitate the

PROGRESS OF THE PUPIL.

······

BY J. WALKER.

·······

A New Edition,

WITH AN APPENDIX

BY W. RUSSELL,

Author of the Companion to every Treatise on Arithmetic.

———▸▶◀◂——

LONDON:

PRINTED FOR JOHN SOUTER, SCHOOL LIBRARY,
73, ST. PAUL'S CHURCH YARD,
By J. and C. Adlard, 23, Bartholomew-Close.

1823.

WITH AN APPENDIX
By W. RUSSELL,
*Author of the Companion to every
Treatise on Arithmetic*

THE

PHILOSOPHY OF ARITHMETIC,

OR,

A COMPLETE ANALYSIS

OF

INTEGERS.

FOR THE USE OF ADVANCED STUDENTS, BUT SUITED
TO THE MOST LIMITED CAPACITY.

Also, an Appendix,

CONTAINING DOMESTIC CALCULATIONS TO BE
PERFORMED MENTALLY.

—o—

BY WILLIAM RUSSELL,

Writing Master and Accountant.

—o—

"Nothing can enter into the affections that stumbles at the threshold."
Blair.

———————

LONDON:

J. SOUTER, SCHOOL LIBRARY,
73, ST. PAUL'S CHURCH YARD.

1826.

Nothing can enter into the affections that stumbles at the threshold.
Blair.

PREFACE.

—o—

THE following Lectures are the result of many years' experience in instructing British youth, of both sexes, in the invaluable science of Arithmetic. They have been repeatedly read by the Author with considerable success in one of the first finishing Establishments in the Metropolis.

Biographical information about him in his books

In *A Companion to Every Treatise on Arithmetic*:

Mr. Russell gives lessons in Writing and Accounts, in Ladies' Schools and Families. He also writes public or private Records in Manuscript, as well as other Documents in which fine Writing may be required.

In *his Philosophy of Arithmetic or a Complete Analysis of Integers*:

The Author has lately written the following Works, which have been introduced into many of the principal Academies in the British dominions, and which may be had of his publisher, Mr. Souter; viz. Companion, Appendix of Walker, Key to ditto, Superior small-hand copies,... Mr. Russell gives Lessons in writing, Arithmetic, Algebra, Book-Keeping, and the Use of the Globes, in Schools and Families.

# 2. What we don't know about Mr. Russell

We could not find a portrait of him.
We do not know when and where he was born
              or when and where he died.
We do not know what education he had or where he learned
mathematics.

3. *A Companion to Every Treatise on Arithmetic* (1821)

The book was not meant to be an independent arithmetic textbook, but just a companion to be used with any textbook to explain the meaning of arithmetic operations and other concepts studied by students.
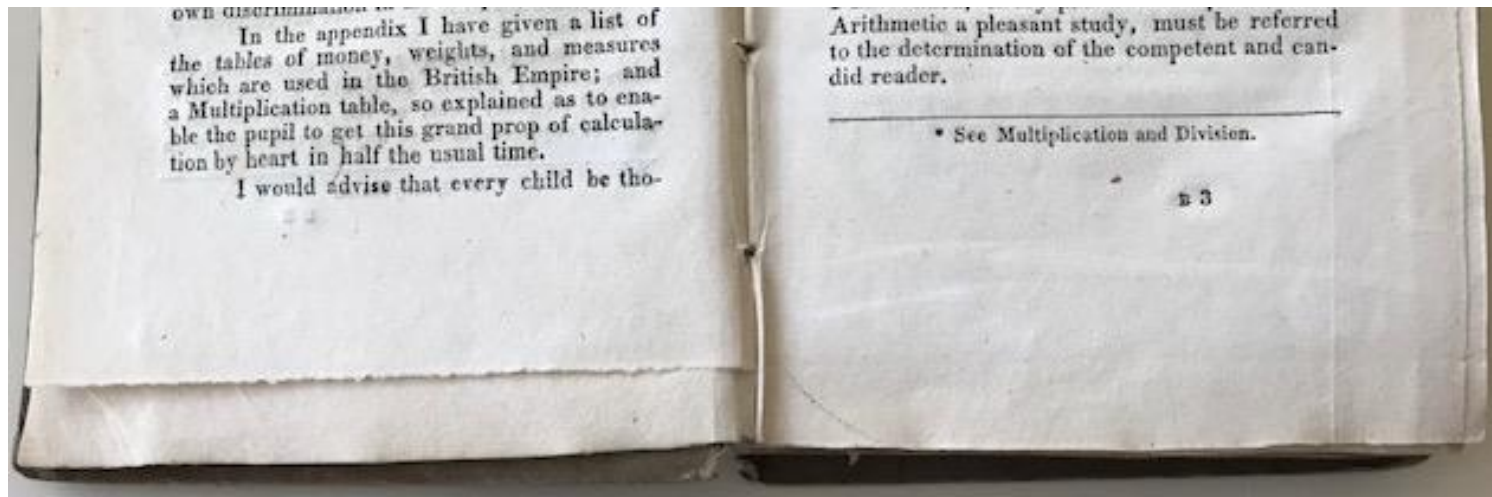
It seems very few copies of the book were printed, and they were distributed only to people who preordered them. (The pages are of different sizes and roughly cut.)

It was meant for both teachers and students, and the content was presented in the form of questions and answers. (It was "catechetical", as in, e.g., Thomas Dilworth's *The Schoolmaster's Assistant*, 1818.)

The concept of the meaning of arithmetic operations is explained in the author's later book *The Philosophy of Arithmetic or A Complete Analysis of Integers*.

The word "philosophy" had a different meaning than it has now.

It was probably used as it was by Newton in his *Mathematical Principles of Natural Philosophy* (1687, 1713, 1726).

What is arithmetic?

(From Chapter I. Introduction)

1. What is arithmetic?
   It is a science, which teaches the art of accounting, and all the powers and properties of numbers.

2. What is Arithmetic considered both as a science and as an art?
   Arithmetic, as a science, explains the properties of numbers; and, as an art, teaches the method of computing by them.

3. How does it appear that Arithmetic is both a science and an art?
   Arithmetic is a science, because its rules are capable of being demonstrated; and it is an art, because it applies those rules to useful calculations, in the concerns of life.

The author wanted to keep the book simple and understandable. And he specifically mentions that this is the reason for not using algebra to explain arithmetic.

He criticizes teachers who, instead of providing logical explanations, tell students just to follow what the teacher is doing, "Read as I read" and "Calculate as I calculate".

There are 49 very short chapters with 354 questions and answers (120 pages total).

The chapters are followed by an Appendix (with 12 more very short chapters) containing tables on money, weight, measures, motion of the earth and sun, paper, quarterly terms, numeration table, and an unusual multiplication table marking numbers in it that are repeated, to help in learning it. At the end are 34 problems for leisure hours and an index.

Selected titles of chapters

II.   Notation and Numeration

IV.   Addition

V.   Subtraction

VI.   Multiplication

VII.   Division

XI.   Addition of fractions

XV.   Proportion of fractions

XVI.   Decimal fractions

XVII.   Addition of decimals

XXIII.   Simple proportion or Rule of three

XXVII.   Simple interest

XXXI.   Brokerage

XXXIII.   Buying and selling of stock

XXXVIII.   Bankruptcy

XLIII.   Alligation or Rule of mixtures

XLVI.   Arithmetical progression

XLVII.   Geometrical progression

XLVIII.   Involution

XLIX.   Evolution

Appendix

Its content: Money, Weight, Measures, Motion of the earth, Paper, Quarterly terms, Numeration table, Multiplication table

Examples from his book

V. SUBTRACTION

27. What is subtraction?
    It is that operation by which we find the difference between two given numbers.
28. What are those numbers called?
    The greater number is called the *minuend*; and the less the *subtrahend*.
29. How do you set down integers to be subtracted?
    I place the subtrahend below the minuend, in the same order as directed in Addition.
30. How do you proceed when any figure in the lower line is greater than its corresponding figure in the upper line?
    I add ten to the upper figure, subtract the lower from the amount; and carry one to the next figure in the lower line.
31. Since you add *ten* to the upper figure, why do you only carry *one* to the next figure in the lower line?
    Because, from the nature of numbers, *ten* in any column, make but *one* in the next column towards the left hand.

XXXV. BARTER

266. What is Barter?

It is the exchanging of one kind of commodity for another, and directs the Merchants and Tradesmen so to proportion the value of their goods that neither party may sustain loss.

267. Give me the rule for calculating questions in Barter?

I first find the value of the commodity whose quantity is given, then I find what quantity of the other, at the rate proposed, may be had for the same money; the result will give the answer.

XLVIII. INVOLUTION.

347. What is the nature of Involution?

    Involution is the method of raising powers from a given number or root.
    Thus, if the first power or root be multiplied by itself, the product is the
    second power or square; this gain by the root, and the product is the
    third power or cube; and if this be gain multiplied by the root, or the
    square multiplied by the square, the product is the fourth power, or
    biquadrate; and so on.

348. What is the exponent, or index of a power?

    The figure that denotes the extent or degree of that power, and exceeds,
    by 1, the number of multiplications used in producing the power; so 2 is
    the exponent or index of the square, 3 the index of the cube, and 4 that
    of the biquadrate, &c.

349. What is the general rule for finding powers?

    I multiply the given number or root continually by itself, till the number of
    multiplications be 1 less than the index of the power to be found.

4. Final remarks

Public schools in the nineteenth century were meant to provide education for working people, and therefore the mathematics that was taught focused mainly on arithmetic skills and their practical use.

But the goals for educating those in upper-middle and higher classes, taught in private schools or by private tutors, were often different.

William Russell's books were meant for upper-middle or higher class audiences. (For example, in problems of counting money, guineas are counted, and not shillings or pounds.)

He wanted his students to understand mathematics. But by understanding he meant *knowing definitions and rules,* and justifying them in terms of *other rules* already established.

We have put the book and these slides on our website at
https://web.nmsu.edu/~pbaggett/ under Notes.

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NEW **A COMPANION TO EVERY TREATISE ON ARITHMETIC,**
IN THE FORM OF QUESTION AND ANSWER

By William Russell

View Book   View Notes

Thank you!

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XXIII. MULTIPLICATION TABLE.*

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12 *a*	14	16 *b*	18 *c*	20 *d*	22	24 *e*
3	6	9	12 *a*	15	18 *c*	21	24 *e*	27	30 *f*	33	36 *g*
4	8	12 *a*	16 *b*	20 *d*	24 *e*	28	32	36 *g*	40 *h*	44	48 *i*
5	10	15	20 *d*	25	30 *f*	35	40 *h*	45	50	55	60 *j*
6	12	18 *c*	24 *e*	30 *f*	36 *g*	42	48 *i*	54	60 *j*	66	72 *k*
7	14 *a*	21	28	35	42	49	56	63	70	77	84
8	16	24 *e*	32	40 *h*	48 *i*	56	64	72 *k*	80	88	96
9	18 *c*	27	36 *g*	45	54	63	72 *k*	81	90	99	108
10	20 *d*	30 *f*	40 *h*	50	60 *j*	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24 *e*	36 *g*	48 *i*	60 *j*	72 *k*	84	96	108	120	132	144

* The inventor of this table is said to be Pythagoras of Samos, hence it is frequently stiled the Pythagoric Abacus.

a 6 times 2 = 12 ; and 4 times 3 = 12 also
b 8 times 2 = 16 ; and 4 times 4 = 16.
c 9 times 2 = 18 ; and 6 times 3 = 18.
d 10 times 2 = 20 ; and 5 times 4 = 20.
e 12 times 2 = 24 ; and 8 times 3 = 24 ; and, 6 times 4 = 24.
f 10 times 3 = 30 ; and 6 times 5 = 30.
g 12 times 3 = 36 ; and 9 times 4 = 36 ; and, 6 times 6 = 36.
h 10 times 4 = 40 ; and 8 times 5 = 40.
i 12 times 4 = 48 ; and 8 times 6 = 48.
j 12 times 5 = 60 ; and 10 times 6 = 60.
k 12 times 6 = 72 ; and 9 times 8 = 72.

It may be of some use to the pupil, while learning the above table, to observe that the product of all the odd numbers in 5 times ends with 5 ; and that of the even numbers with a cipher ; or 5 times any number is just half of 10 times the same: the product of any number by 10 is just that number with a cipher annexed ; and the product of any of the nine digits by 11 is just two of them, the one considered as the units' place of the product, and the other as the tens'.

Although I do not approve of difficult calculations being put into the hands of beginners, yet, as it is not unlikely that this small work may meet the eye of those that are *advanced* in Arithmetic, I have, therefore, given a few amusing questions for exercise in leisure hours, which are as follows:—

I.

On Christmas week a Bond-street blade
    Ran through one-third his income;
Besides ten pounds he gave to Ned—
    Ned bowed, and said 'twas handsome.
With heavy heart, this spendthrift elf
    Went home to count his cash, sir,
He found but half his income left,
    With fifteen pounds to dash sir.
        What was his income?   *Ans.* £150.

2.

As I was beating on the forest grounds,
Up starts a hare before my two greyhounds;
The dogs, being light of foot, did quickly run
Unto *her* fifteen yards just twenty-one,
The distance that she started up before
Was four-score sixteen yards just, and no more;
Now, I would have you unto me declare
How far they ran before they caught the hare?
    Ans. the dogs ran 336 yards and the hare
        240 yards.

3.

A draper sold cloth at 11s. per yard, by which he cleared $\frac{3}{8}$ of his money; he afterwards raised it to 13s. 9d. per yard; what did he clear per cent by the latter price?

Ans. 100 per cent.