Tactile drawings and 3-D objects: Two keys to geometry for a blind student in an inclusion university course for preservice K-8 teachers

TSG 4: Mathematics education for students with special needs
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Plan of talk

- Introduction and background
  - Office of Assistive Technologies
  - Tools for the student
- Challenges
- Examples of five lessons that were adapted for the student
- Final remarks and Lessons learned

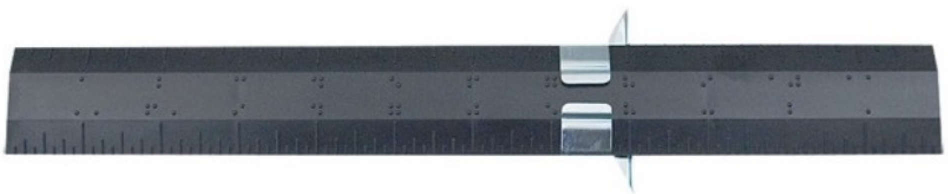
Introduction and background

One week before the start of the 2019 spring semester I got a call from the university Assistive Technologies office:
"A congenitally blind student is enrolled in your geometry class."

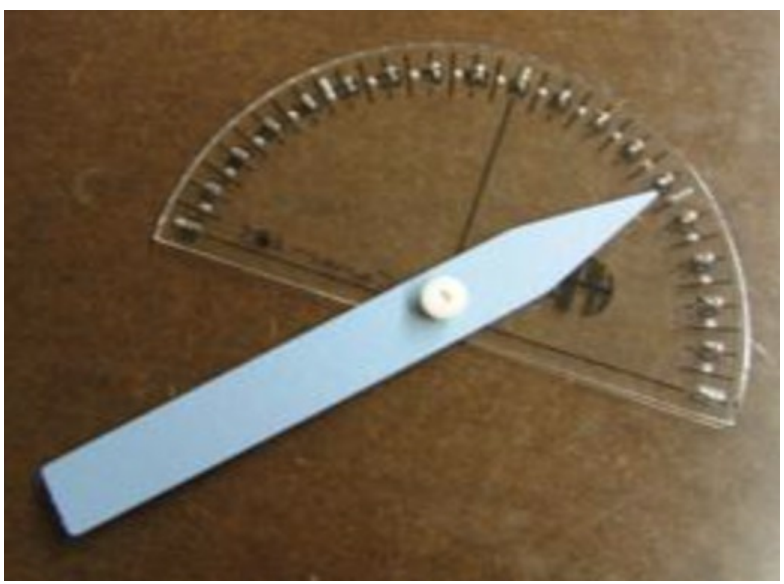
(It is not a proofs class, but an active learning hands-on class involving rulers, protractors, compasses, scissors, and designing and putting together many objects.)

What to do?

I immediately arranged to meet with the university Assistive Technologies specialist, and I brought her a copy of my course pack for the class.



She loaned me a braille ruler and protractor for the student.



She said that the student would bring her own braille note-taker and talking calculator to the class.

Challenges

- How to make the course materials accessible to the student (aka Joslyn) so she could be successful

- How to create a classroom environment for the sighted students to be fully engaged

To the rescue

My department assigned a math grad student to sit with Joslyn during the class (as part of his assistantship). (I refer to him as Mr. E.)

From Assistive Technologies
A machine that takes as input an electronic text in Word and outputs the text on paper in braille (but it only handles text, no pictures)

The course materials were filled with diagrams of 2-D and 3-D nets, still a big problem….

To the rescue again

The availability of a PIAF machine (Picture in a Flash)

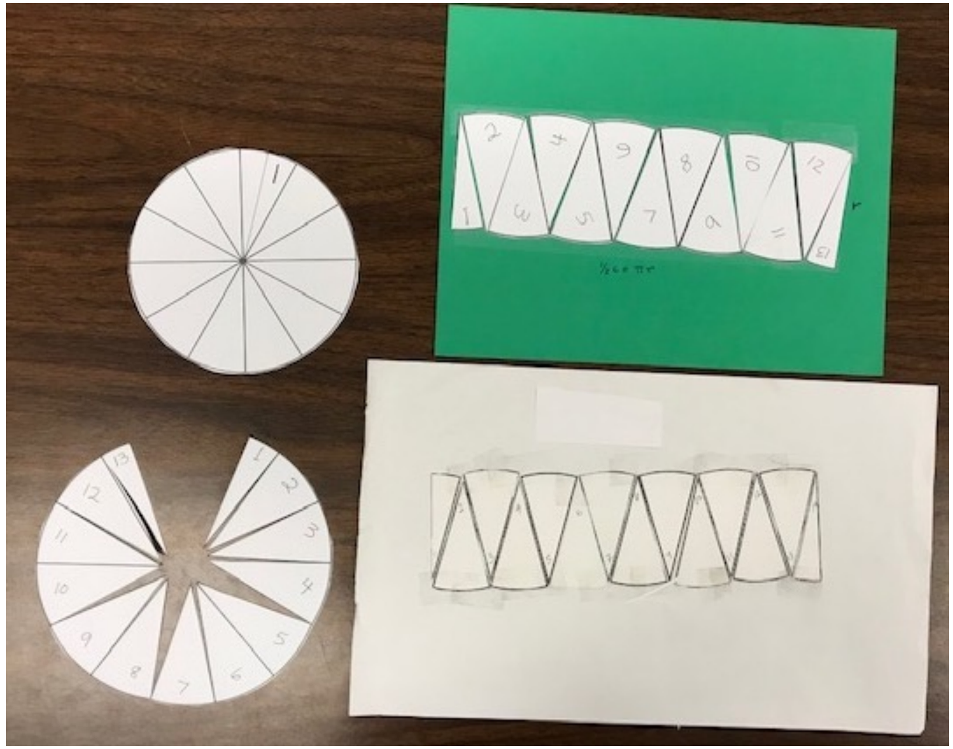
The PIAF uses “capsule paper” or “swell paper”. A diagram can be printed onto the capsule paper using a standard printer, and then the paper is passed through the machine, which heats it, and the areas where the paper has been printed on swell to create ridges and textures, namely, tactile drawings which can be perceived by touch.

So some equipment for Joslyn is available, and she can take notes in class (and submit her text assignments electronically). And she sits next to a sighted assistant.

But what about the materials to be studied in class?

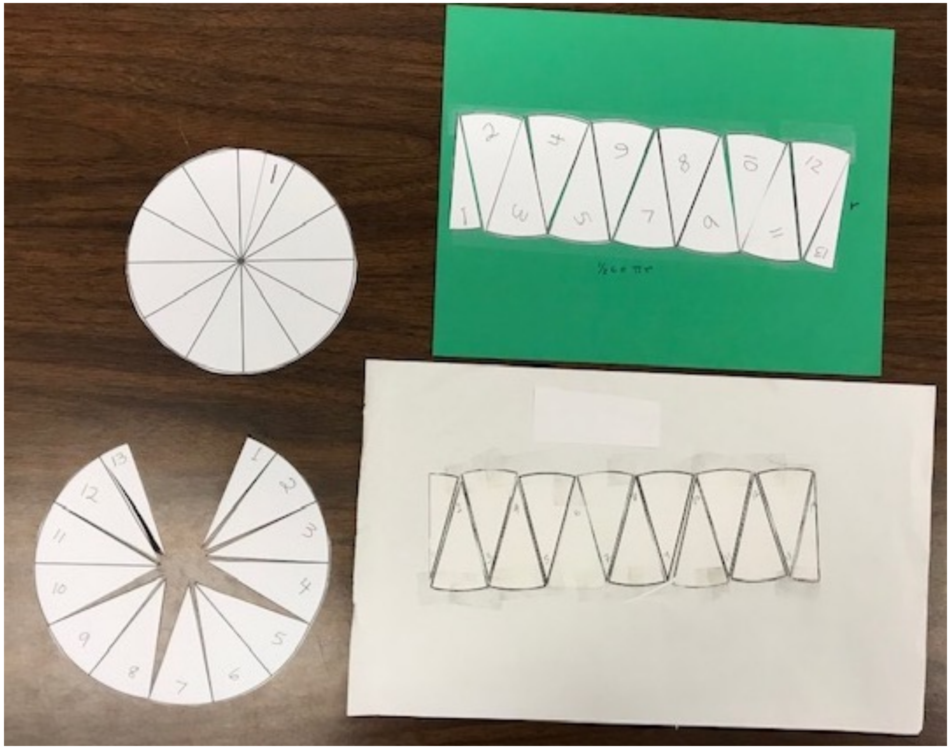
Examples of lessons that were adapted for the student

We'll look briefly at five units



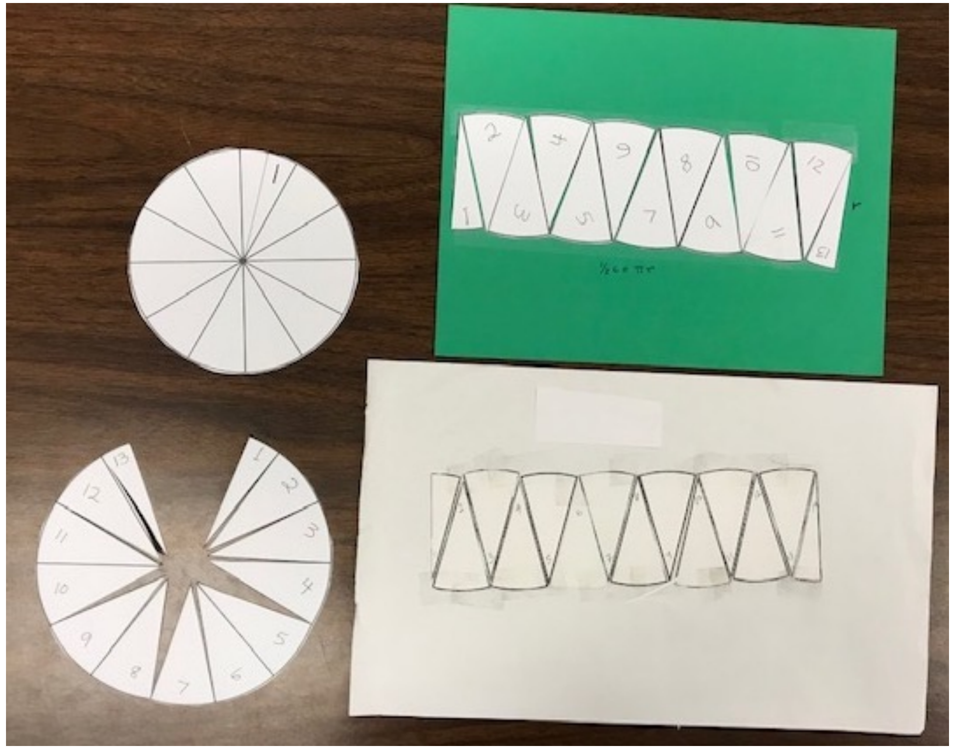
Area of a circle using sectors

In this unit we did not yet have PIAF-made illustrations.
Earlier in the course we had rolled cardboard circles along a straight line to get their circumferences, and divided those numbers by the circle’s diameters, getting an estimate for π. So we knew that the formula for the circumference of a circle was π*d, or 2πr. In this unit the formula for the area of a circle, $A = \pi * r^2$, was informally derived.



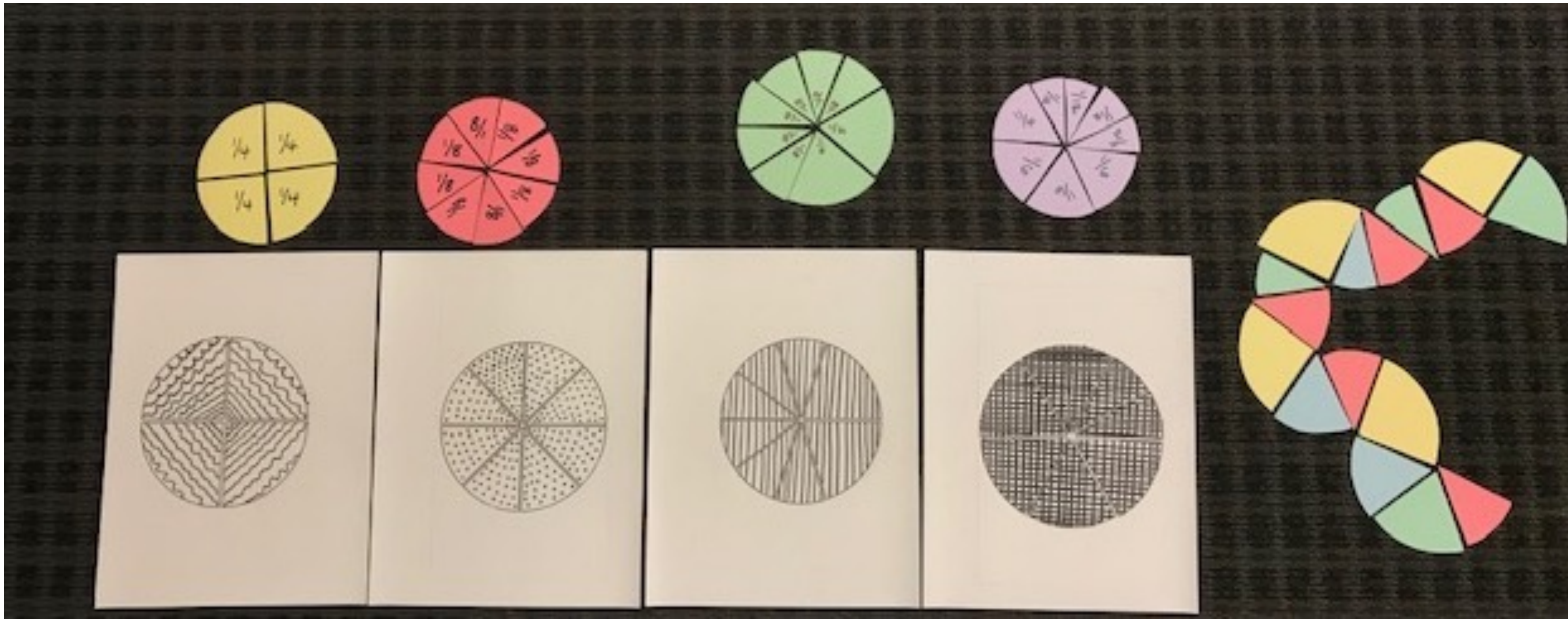
Area of a circle using sectors

Students were given circles on cardstock divided into sectors. They cut the sectors as shown and rearranged them into a lumpy rectangle whose height is r and whose length is half of the circle's circumference, or $\pi * r$. So its area is $\pi * r^2$, which is thus also the area of the circle.



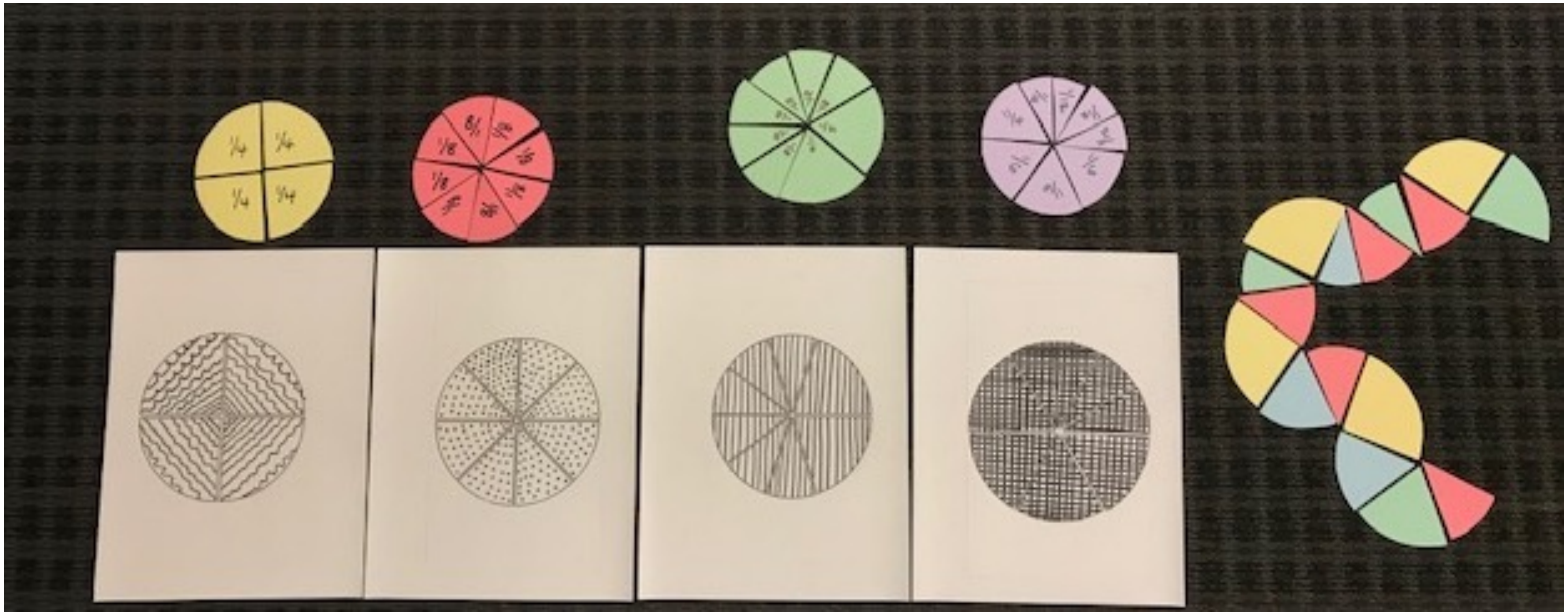
Area of a circle using sectors

Joslyn’s work is shown in the picture on the lower right. She was given the circle from cardstock, with radii “scored” lightly with a compass point so that she could feel them. Mr. E cut the radii, and she carefully tore the sectors apart and reassembled them into the rectangle. She wrote about this unit in an assignment and said that she was surprised at such an interesting way to get the formula for the area of a circle.



Fractions of a circle AKA Caterpillar

Students got 4 index cards of different colors. Using compasses and protractors, they drew and cut out 4 circles, each 5 inches in diameter. One has 4 quarters; one is cut into 8ths ; one has 4 6ths and 4 12ths; and the last has 2 5ths and 6 10ths (see picture). Joslyn was given these sectors, cut out of poster board, together with 4 tactile drawings with textured areas as shown.



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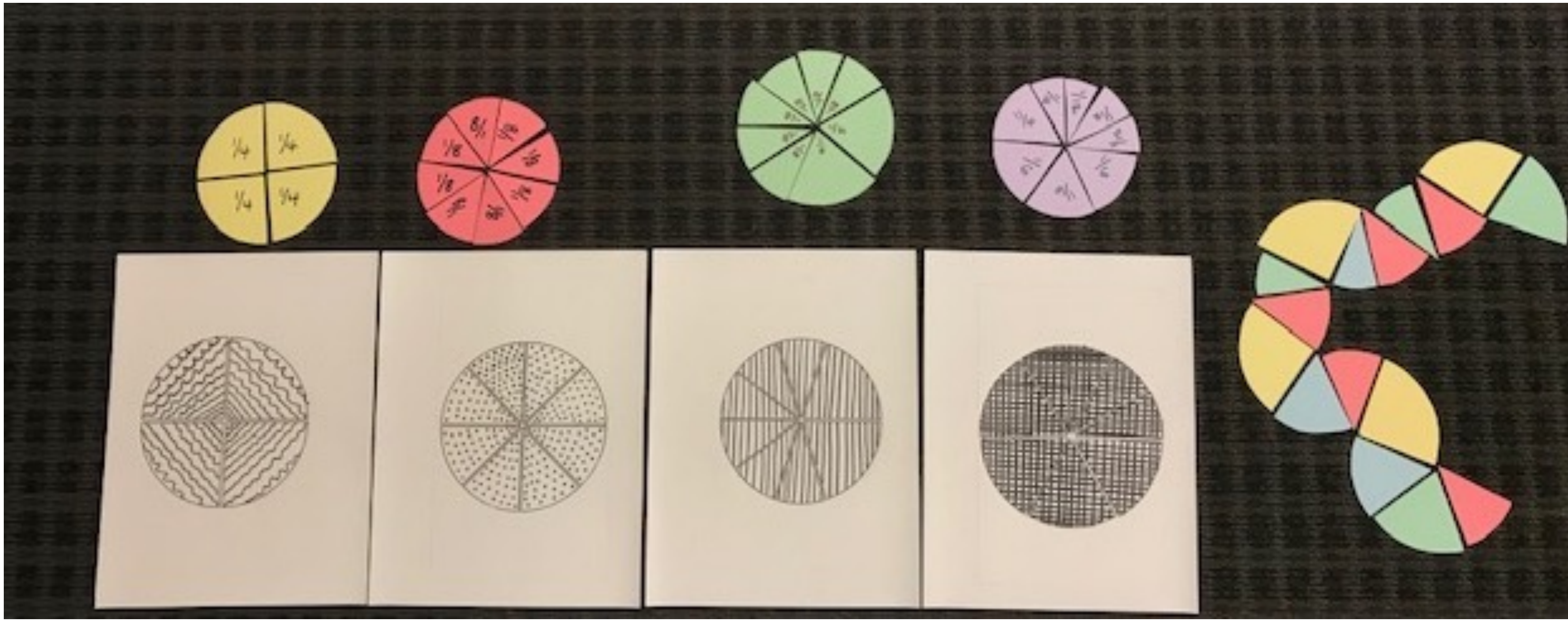

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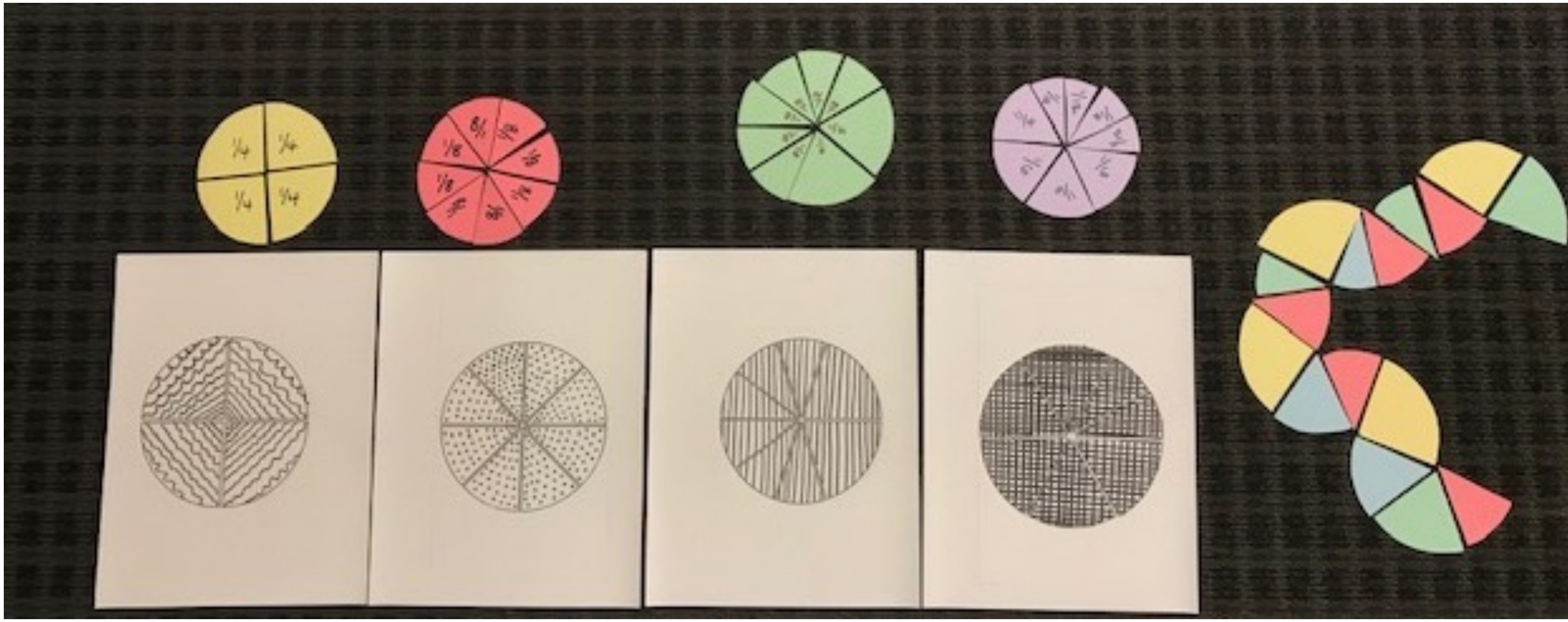
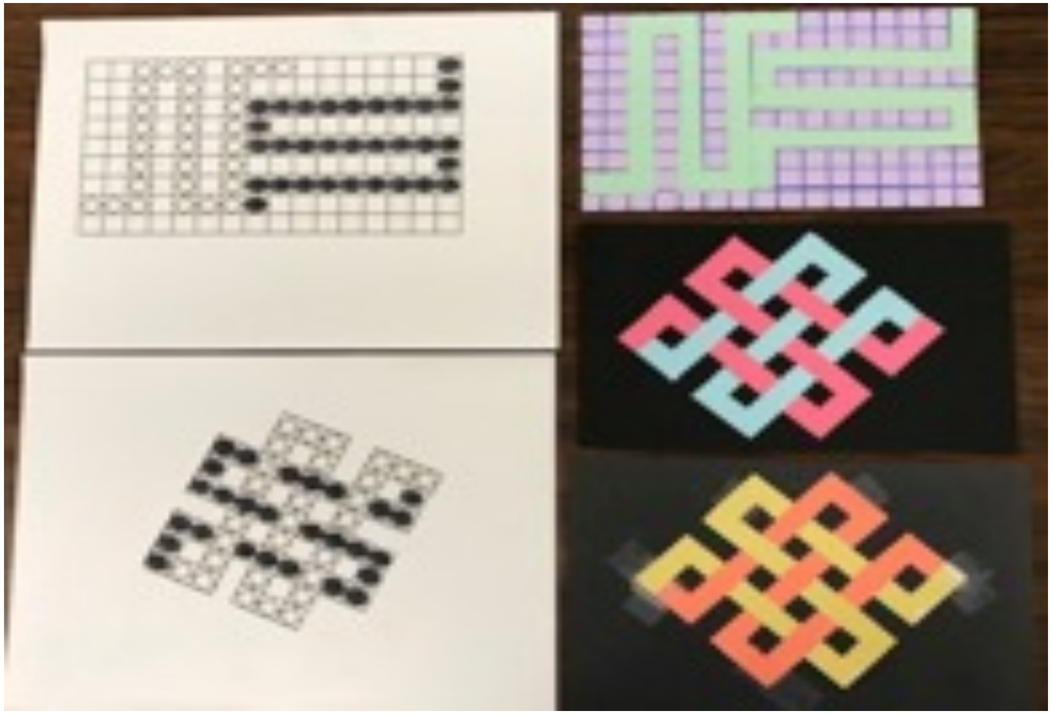
The tasks included trying to make circles with different colored sectors, adding the degrees in the sectors to see if they totaled 360 (or adding the fractions to see if they summed to one), and at the end, making "caterpillars" with sectors of different shapes.



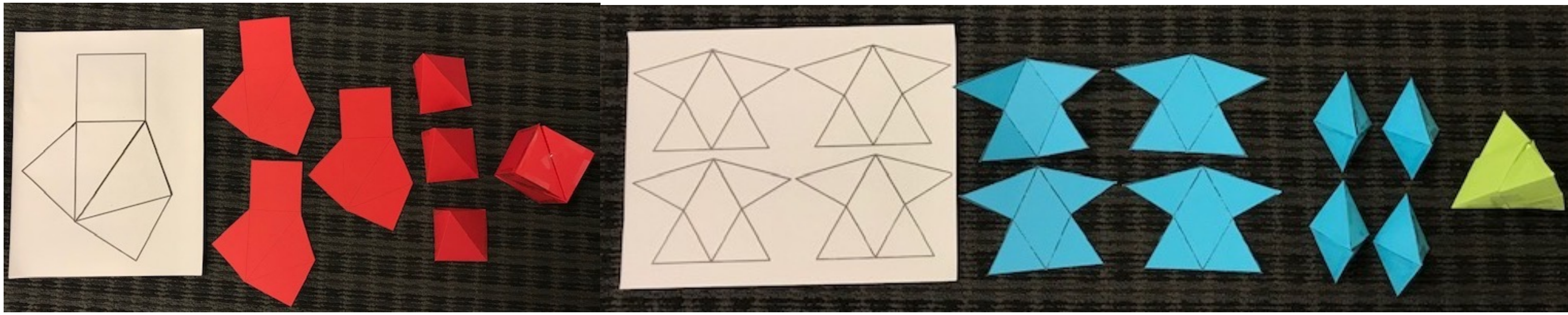
Fractions of a circle AKA Caterpillar

In the evaluation at the end of the semester Joslyn particularly mentioned this unit as being useful and interesting. She said she used her protractor to measure the angles, and she was able to add fractions and sum the degrees in the sectors using her talking calculator. She also said she enjoyed manipulating the pieces of poster board and making a caterpillar at the end.



Ancient Knot

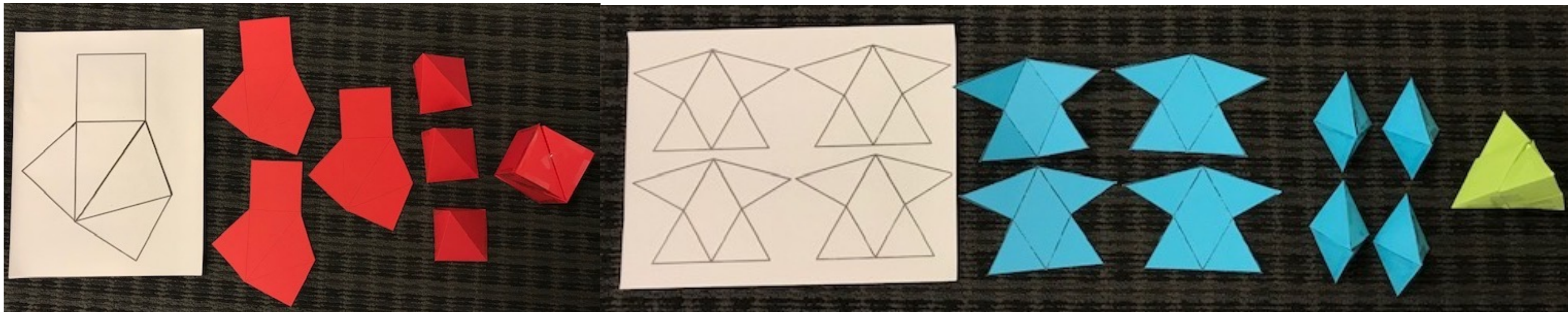
In this unit, also at Ancient Knot, students divide an 8 in. by 5 in. index into .5 inch x .5 inch squares. They copy the pattern shown on the above right onto their card and cut out two "camel" shapes. They exchange one with a neighbor to get two different colors. These can be woven into an "ancient knot" with no beginning and no end. Two tactile drawings given to Joslyn are shown on the left. The instructor's knot is in the middle on the right. Mr. E cut out the camels for her, and she wove them together. Her completed knot is at the bottom right. Another unit involving weaving, Julekurv AKA Valentine Basket, was also adapted for Joslyn with tactile drawings.



A cube into 3 congruent pyramids, and a tetrahedron into 4 congruent polyhedra

In the first unit (on the left), 3 congruent right pyramids make a cube. Mr. E cut out from card stock and scored 3 2-D nets for the pyramids. (Scoring made folding easier.) Joslyn also had a tactile drawing for one net, as shown. Her task was to fold and tape together each of the nets, and to combine them into a cube.

This task was tricky for the sighted students (who also had to cut out and score the 2-D patterns). But there was a remarkable outcome: Joslyn was among the first in the class to complete the task.



A cube into 3 congruent pyramids, and a tetrahedron into 4 congruent polyhedra

In the 2nd unit (on the right), 4 congruent polyhedra make a tetrahedron. As before, Joslyn's task was to fold and tape together the pieces and form them into the tetrahedron.

Most sighted students could not make the tetrahedron without help, but Joslyn did it alone after some effort. She indicated that lessons like these, involving 3-D objects, were her favorites.

See also (at the website for the course https://web.nmsu.edu/~pbaggett/) Pretty gift box, Four cubes, and Cutting Polyhedra, which were adapted for her.

Final remarks and Lessons learned

A blind student can actually participate successfully in a hands-on university geometry class, with the help of Braille notes and good-quality tactile drawings (a huge shout out to Carol Walker and her Assistive Technologies crew!) and 3-D objects and their 2-D nets, allowing her to "see" with her hands (Healy et al., 2013; Riyzuer et al. (2004); Shimomura et al. (2013).).

It takes a great deal of daily planning and help from outside. But it is not necessary to tailor the course specifically for the blind person if some supports are available and in place. (But Mr. E, Joslyn's partner, had to learn to help only when asked, and not to do the task for her. Often she would say, "I can do this. Let me do it!") When Mr. E was absent, many students would volunteer to be Joslyn's partner.

Anonymous evaluations at the end of the course from sighted students indicate Joslyn's presence actually enriched the class substantially. At sessions in which tactile drawings were used, they were shown to students on the overhead projector. Students remarked that the course made them "see" geometry in a new light.

References

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Slides from this talk will be available at
https://web.nmsu.edu/~pbaggett/notes/notes.html
under the title
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Thank you!

Note. A shorter version of this talk was presented at the Joint Math Meetings in Denver, CO, January 2020 under the title An Approach to Accommodating a Congenitally Blind Student in an Inclusive University Geometry Course for Preservice K-8 Teachers.