Primes in their prime

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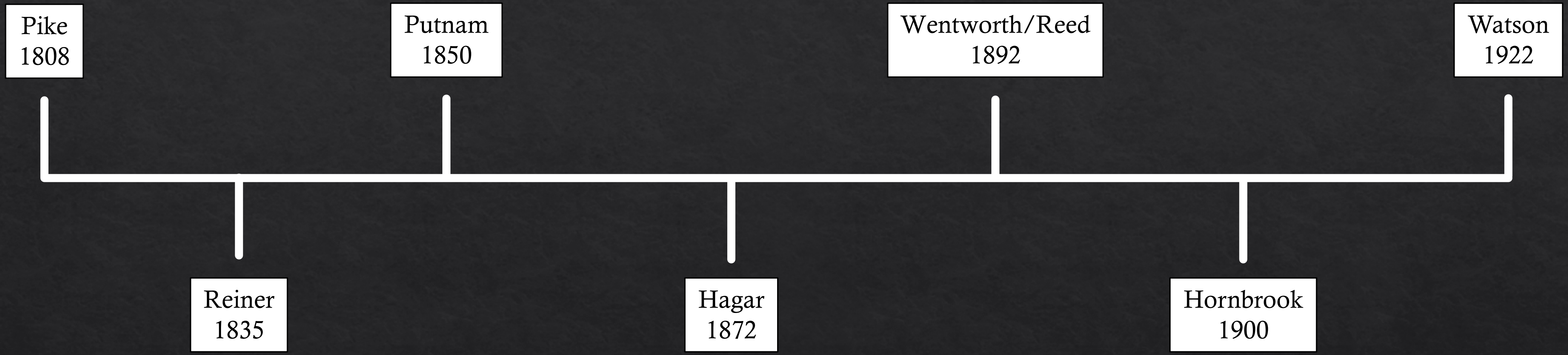
Introduction

- Prime numbers are an ancient concept, with some of the earliest uses ranging from 1550 BC to 300 BC
- As one of the more ancient mathematical concepts, mathematicians have played around with primes for centuries
- There are a host of theorems dealing with primes in number theory, algebra, cryptography, etc.

Big Question

- Restricting our focus to the USA, what was the treatment of primes in the 19th century?
- Elementary/middle school level
- Looking at 7 books
- Dating between 1808 and 1922
- How are primes defined?
- What types of questions are asked?
- The word choice and exercises will be important!

Timeline


Pike 1808
Reiner 1835
Putnam 1850
Hagar 1872
Wentworth/Reed 1892
Hornbrook 1900
Watson 1922

A New Complete System of Arithmetick (Pike, 1808)

- The definition of the prime was missing from this text
- But it does have an important rule

7. If a number cannot be divided by ſome number leſs than the ſquare root thereof, that number is a *prime*.

8. All *prime* numbers, except 2 and 5, have 1, 3, 7, or 9 in the place of units; and all other numbers are *compoſite*.

A New Complete System of Arithmetick (Pike, 1808)

- An interesting "real-world" question:

1. How many barley corns will reach from Newburyport to Boston, it being 43 miles?

```
     43  miles.
      8
   ----
    344 furlongs.
     40
  -----
  13760 rods.
     5½
  -----
  68800
   6880
  -----
  75680 yards.
      3
  -----
 227040 feet.
     12
 ------
2724480 inches.
      3
 ------
8173440 Answer.
```

```
 3)8173440 proof.
   -------
12)2724480
   -------
  3)227040
   -------
  11)75680
   -------
      6880
         2
   -------
4|0)1376|0
   -------
     8)344
      ----
        43
        --
```

Here I divide by 11, and multiply the quotient by 2, because twice 5½ is 11; or I might first have multiplied by 2, and, then, have divided the product by 11.

Lessons on Number (Reiner, 1835)

- This book is written in a call-and-response way
- Definition is in steps:

Teacher. If a number contains another number exactly, what may be said of that number?

Pupils. The first number is *divisible* by the second.

T. Give an instance.

P. 20 is divisible by 10.

Teacher. Find the divisors of the numbers 11, 12, and 13.

Pupils. Divisors of 11 are 1 and 11.

" 12 are 1, 2, 3, 4, 6, and 12.

" 13 are 1 and 13.

Lessons on Number (Reiner, 1835)

T. What is there to be remarked as to the number of divisors?

P. Some numbers have more divisors than others.

T. Which is the least number of divisors a number can have?

P. Two divisors; 1 and the number itself.

T. Such numbers are called *prime* numbers. Find 10 numbers which are *prime* numbers.

P. 1, 2, 3, 5, 7, 11, 13, 17, 19, 23.

T. Is 14 a prime number?

P. No, because its divisors are 1, 2, 7, and 14.

Lessons on Number (Reiner, 1835)

Answers to the Exercises.

1. A number having no other divisors besides unity and the number itself, is called a prime number.

2. The prime numbers are —

1.	17.	43.	73.	101.
2.	19.	47.	79.	103.
3.	23.	53.	83.	107.
5.	29.	59.	89.	109.
7.	31.	61.	91.	113.
11.	37.	67.	97.	119.
13.	41.	71.		

3. Numbers which, multiplied together, produce a given number, are said to be factors of that number.

4. Prime numbers which, multiplied together, produce a given number, are said to be prime factors of that number.

5. $60 = 2 \times 2 \times 3 \times 5.$
$66 = 2 \times 3 \times 11.$
$70 = 2 \times 5 \times 7.$
$74 = 2 \times 37.$
$80 = 2 \times 2 \times 2 \times 2 \times 5.$
$85 = 5 \times 17.$
89 is a prime number.
$90 = 2 \times 3 \times 3 \times 5.$
$100 = 2 \times 2 \times 5 \times 5.$
$102 = 2 \times 3 \times 17.$
$115 = 5 \times 23.$
$200 = 2 \times 2 \times 2 \times 5 \times 5.$
$300 = 2 \times 2 \times 3 \times 5 \times 5.$
$400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5.$
$500 = 2 \times 2 \times 5 \times 5 \times 5.$

The American Common-School Arithmetic (Putnam, 1850)

71. A *Unit* is any single thing; as one, one hat, one school, one nation.

An *Integer* is any whole number; as 1, 8, 15.

An *Even Number* is one whose right hand figure is 0, 2, 4, 6, or 8.

An *Odd Number* is one whose right hand figure is 1, 3, 5, 7, or 9.

A *Composite Number* is one that is composed of two or more factors. (**25** and **28.**)

A *Prime Number* is one that has no factors except itself and unity. Numbers are *prime to each other* that have no common factor.

Thus, 8 and 15, though composite numbers, are prime to each other, for $8 = 2 \times 2 \times 2$, and $15 = 3 \times 5$.

A Prime Factor of a number is a prime number that will divide it without a remainder. Thus, 2, 3, and 5, are prime factors of 30.

EXAMPLES FOR PRACTICE.

1. How many times is 4 contained in 3416.8?

$$4\,\overline{)3416.8}$$
$$854.2$$

The American Common-School Arithmetic (Putnam, 1850)

- Example questions:

3. Is 25 a composite or a prime number? Is 32? 37? 19? 22? 63? 51?

4. Are 9 and 12 prime to each other? Are 10 and 12? 7 and 15? 16, 24, and 25?

5. What are the prime factors of 6? Of 8? 9? 10? 11? 12? 15? 18? 23? 45?

The American Common-School Arithmetic (Putnam, 1850)

RULE FOR FINDING ALL THE PRIME FACTORS OF A NUMBER.

Divide the number by any one of its prime factors; then that quotient by another, and so on till the quotient is a unit. The several divisors are all the prime factors of the number.

7. What are the prime factors of 1260?

Ans. 2, 2, 3, 3, 5, 7.

2	1260
2	630
3	315
5	105
7	21
3	3
	1

We see that *a number is equal to the product of all its prime factors;* for $2 \times 2 \times 3 \times 3 \times 5 \times 7 = 1260$.

NOTE. When a factor is repeated, it may be written but once, by placing a small figure above it at the right hand, called an *index*, to indicate how many times it is used as a factor. Thus, instead of writing $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$, we may write $2^3 \times 3^2 \times 5 = 360$; $5 \times 5 \times 5 \times 5 \times 2 \times 2 \times 3 \times 3 \times 3 = 67500$, may be written $5^4 \times 2^2 \times 3^3 = 67500$.

The American Common-School Arithmetic (Putnam, 1850)

The following truths will aid the pupil in finding the prime factors and other divisors of numbers.

2 will divide all even numbers.

3 will divide all numbers the sum of whose figures is divisible by 3. Thus, 684 is divisible by 3, because $6+8+4=18$ is divisible by 3.

4 will divide any number whose two right hand figures are divisible by 4.

5 will divide any number whose right hand figure is 0 or 5.

6 will divide any even number that is divisible by 3.

8 will divide any number whose three right hand figures are divisible by 8.

9 will divide any number the sum of whose figures is divisible by 9.

10 will divide any number whose right hand figure is 0.

11 will divide any number in which the sum of the figures in the odd places is equal to the sum of the figures in the even places; or in which their sums differ by a number which can be divided by 11 without a remainder. Thus, 85426 is divisible by 11, because $8+4+6=5+2+11$. So is 63561894; because $6+5+1+9=3+6+8+4$.

12 will divide any number which is divisible by 3 and 4, because

A number that contains two or more factors that are prime to each other, is divisible by the product of those factors.

7, 11, and 13, are factors in numbers of 4 places in which two similar figures enclose two naughts; as 1001, 5005, 10010, 50050, &c.

For a more extended list of factors, with their signs or marks of recognition, see a little work entitled "The Plain Calculator, by Lewis Joerres, Professor of Mathematics from Prussia."

A Common School Arithmetic (Hagar, 1872)

96. The **Factors** of a number are the integers which being multiplied together will produce that number.

Thus, 2, 3 and 5 are the factors of 30; for $2 \times 3 \times 5 = 30$.

97. A **Prime Number** is an integer that has no factor except itself and 1.

Thus, 1, 3, 5, 7 and 11 are prime numbers.

99. A **Prime Factor** is a factor which is a prime number.

The prime factor 1 is not commonly mentioned, since it is a factor of every integer.

Numbers are said to be *mutually prime*, or *prime to each other*, when they have no common factor except 1.

A Common School Arithmetic (Hagar, 1872)

What are the prime factors of—

1. 75? *Ans.* 3, 5, 5.
2. 144? *Ans.* 2^4, 3^2.
3. 116?
4. 340? *Ans.* 2^2, 5, 17.
5. 180?
6. 3809? *Ans.* 13, 293.
7. 99? *Ans.* 3^2, 11.
8. 231? *Ans.* 3, 7, 11.
9. 875?
10. 1110? *Ans.* 2, 3, 5, 37.
11. 4004?
12. 6783? *Ans.* 3, 7, 17, 19.

What are all the different factors or divisors of—

13. 70? *Ans.* 2, 5, 7, 10, 14, 35 and 70.
14. 30?
15. 56? *Ans.* 2, 4, 7, 8, 14, 28 and 56.
16. 42? *Ans.* 2, 3, 6, 7, 14, 21 and 42.
17. 63?
18. 105? *Ans.* 3, 5, 7, 15, 21, 35 and 105.

19. How many of the different factors or divisors of 100 are prime, and how many are composite?

Ans. Three are prime and six are composite.

20. How many of the different factors or divisors of 210 are prime, and how many are composite?

109.—1. What are the FACTORS of a number? What is a prime number? Name some number that is the product of two prime numbers. What is a composite number? Give an example of a composite number.

2. What is a PRIME FACTOR? Name a prime factor of 30. A composite factor of 30. When are two numbers prime to each other?

The First Steps in Number (Wentworth/Reed, 1892)

Name a number that will divide six without a remainder.
(2.)
Two is a *factor* of six.

What are the factors of seven? three? eleven? five? thirteen? seventeen? twenty-three? twenty-nine? thirty-one? forty-one?

Name another number of which the factors are simply *one* and the number itself; another; another.

Beginning with one, write in order a dozen numbers whose factors are simply one and the number itself.

Describe these numbers with reference to their factors.

All such numbers are prime numbers.

The First Steps in Number (Wentworth/Reed, 1892)

Complete this work:

The prime factors of 40 =
The prime factors of 42 =
The prime factors of 45 =
The prime factors of 44 =
The prime factors of 48 =
The prime factors of 49 =
The prime factors of 50 =
The prime factors of 54 =
The prime factors of 56 =
The prime factors of 60 =
The prime factors of 63 =
The prime factors of 64 =
The prime factors of 66 =
The prime factors of 70 =
The prime factors of 72 =

Grammar School Arithmetic (Hornbrook, 1900)

31. A number that has no integral factors except itself and 1 is called a **Prime Number**. Think of each of the numbers from 2 to 10 and tell which of them are prime.

2 is the first prime number, as 1 is considered neither prime nor composite.

Grammar School Arithmetic (Hornbrook, 1900)

36. Divide 13.5 by the 2d prime number.

37. Divide the 3d prime number by .8.

38. Find the difference between 26.4 and the 11th prime number.

Grammar School Arithmetic (Hornbrook, 1900)

	11	~~21~~	31	41	~~51~~	61	71	~~81~~	~~91~~
2	~~12~~	~~22~~	~~32~~	~~42~~	~~52~~	~~62~~	~~72~~	~~82~~	~~92~~
3	13	23	~~33~~	43	53	~~63~~	73	83	~~93~~
~~4~~	~~14~~	~~24~~	~~34~~	~~44~~	~~54~~	~~64~~	~~74~~	~~84~~	~~94~~
5	~~15~~	~~25~~	~~35~~	~~45~~	~~55~~	~~65~~	~~75~~	~~85~~	~~95~~
~~6~~	~~16~~	~~26~~	~~36~~	~~46~~	~~56~~	~~66~~	~~76~~	~~86~~	~~96~~
7	17	~~27~~	37	47	~~57~~	67	~~77~~	~~87~~	97
~~8~~	~~18~~	~~28~~	~~38~~	~~48~~	~~58~~	~~68~~	~~78~~	~~88~~	~~98~~
~~9~~	19	29	~~39~~	~~49~~	59	~~69~~	79	89	~~99~~
~~10~~	~~20~~	~~30~~	~~40~~	~~50~~	~~60~~	~~70~~	~~80~~	~~90~~	~~100~~

Modern Intermediate Arithmetic (Watson, 1922)

A number that has no fractional part is an **integer**; *e.g.* 6, 9, 21, 14.

A number that is composed of an integer and a fraction is a **mixed number**; *e.g.* $3\frac{1}{2}$, $11\frac{2}{3}$.

In speaking of the factors of a number, we usually refer only to factors that are integers; though a fraction or a mixed number may be a factor.

A number that has no factor but itself and 1 is a **prime number**; *e.g.* 1, 2, 3, 5, 7, 11.

Modern Intermediate Arithmetic (Watson, 1922)

Oral

1. Classify these numbers by telling whether they are prime or composite: 2, 7, 11, 6, 9, 15, 19, 16, 18, 27.

2. What even number is prime?

3. Which of the odd numbers given in question 1 are composite?

4. Which of the even numbers are composite?

5. Name all the prime numbers smaller than 50.

6. Give the prime factors of 4; of 9; of 12; of 21; of 30.

Final Thoughts

- Integers weren't consistently included in the definition of primes
- Most of the questions were the same (i.e. what are the prime factors of…)
- Some slight difference in conceptual questions (i.e. what is the only even prime?)

	Is 1 prime?	**Integers in definition?**	**Sieve of Eratosthenes?**
Pike (1808)	Unknown	Unknown	Unknown
Reiner (1835)	Yes	No	No
Putnam (1850)	Yes	No	No
Hagar (1872)	Yes	Yes	No
Wentworth/Reed (1892)	Yes*	No	No
Hornbrook (1900)	No	Yes*	Yes
Watson (1922)	Yes	Yes*	No

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