# DENOMINATE NUMBERS IN AMERICAN SCHOOL TEXTBOOKS FROM 1831 to 1900

# INTRODUCTION

- In this talk the concept of denominate numbers in several American school textbooks from 1831 to 1900 will be examined.

# Books in this study

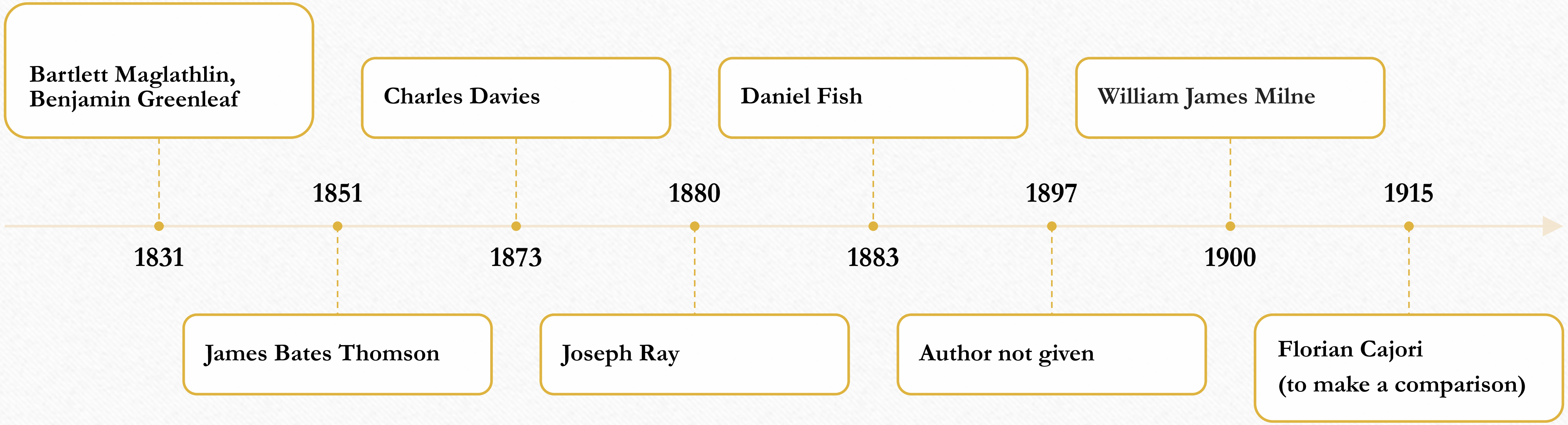
- *The Complete Arithmetic* by Henry Bartlett Maglathlin and Benjamin Greenleaf, 1831.
- *Higher Arithmetic, Or The Science and Application of Numbers* by James Bates Thomson, 1851
- *Practical Arithmetic* by Charles Davies, 1873
- *Ray's New Higher Arithmetic* by Joseph Ray, 1880
- *Fish's Arithmetic Number One[-two]* by Daniel Fish, 1883
- *A Complete Practical Commercial Arithmetic*, Author not given, 1897
- *Intermediate Arithmetic* by William James Milne, 1900

and one extra book to use as a comparison:

- *School Arithmetics Grammar School Book*, Florian Cajori, 1915.

This needs work. The books should be correctly placed on the scale.

# Timeline



# Definitions

- A **denomination** is the name of a unit of measure of a concrete number. A **denominate number** is a concrete number which expresses a particular kind of quantity; as 5 feet, 9 pounds.

Ray, 1880.

# Definitions

- A number used in connection with some specified thing is called a **Concrete Number.** Examples : 6 men, 11 inches, 4 feet etc….
- A number used without reference to any particular thing is called an **Abstract number.** Examples: 1, 2, 3, 4, etc.…

Milne, 1900.

# Definitions

- A concrete number in which the unit of measure is established by law or custom is called a **Denominate Number.** Examples: 5 gallons, 8 rods, etc.
- A denominate number which is composed of units of one denomination only is called a **Simple Denominate Number.** Examples: : 5 gallons, 3 feet, etc.
- A denominate number which is composed of units of two or more denominations that are related to each other is called a **Compound Denominate Number.** Example: 5 yards 2 feet 8 inches.

Milne, 1900.

Compound Denominate Numbers are sometimes called **Compound Numbers.**

# Measures of Denominate Numbers

Ray (1880) (1)Value (2)Weight (3) Extension (4)Time

Fish (1883) (1) Extension (2)Weight (3) Capacity (4)Time

**Extension** is that property of matter by which it occupies space.

# What I looked for in these books

- Treatment of denominate numbers in general from 1831 to 1900, and how Florian Cajori's 1915 book compares.
- Specifically, in the 19$^{th}$ century books, in what cases can multiplication and division of denominate numbers be performed? For example, can multiplication of denominate numbers by numbers of the same or different denominations be performed?

In the books, arithmetic operations are preceded by denominate numbers, and denominate numbers are presented as:

- Reduction of denominate numbers (Descending, Ascending)
- Addition of denominate numbers
- Subtraction of denominate numbers
- Multiplication of denominate numbers.
- Division of denominate numbers.

Rules for reduction are the same in all the textbooks.

Ray, 1880.

REDUCTION OF COMPOUND NUMBERS.

218. **Reduction of Compound Numbers** is the process of changing them to equivalent numbers of a different denomination.

Reduction takes place in two ways:

*From a higher denomination to a lower.*
*From a lower denomination to a higher.*

PRINCIPLES.—1. *Reduction from a higher denomination to a lower, is performed by multiplication.*

2. *Reduction from a lower denomination to a higher, is performed by division.*

# Some reduction exercises

**Davies, 1873.**

4 farthings(qr or far) = 1 penny (d)
12 pence (d) = 1 shilling (s)
20 shillings (s) = 1 pound (£)



**189.** Reduction Descending is the operation of changing a number from a greater unit to a less.

**190.** Reduction Ascending is the operation of changing a number from a less unit to a greater.

## 191. Reduction Descending

1. Reduce £27 6s. $8\frac{1}{2}$d., to the denomination of farthings

Analysis.—Since there are 20 shillings in £1, in £27 there are 27 times 20 shillings, or 540 shillings, and 6 shillings added, make 546s. Since 12 pence make 1 shilling, we next multiply by 12, and then add 8d. to the product, giving 6560 pence. Since 4 farthings make 1 penny, we next multiply by 4, and add 2 farthings to the product, giving 26242 farthings for the answer: Hence, the following

OPERATION.

£27 6s. 8d. 2far.
20
546s.
12
6560d.
4
26242, *Ans.*

# Reduction Ascending

**Davies, 1873**

4 farthings (qr or far) = 1 penny (d)

12 pence (d) = 1 shilling (s)

20 shillings (s) = 1 pound (£)

## 192. Reduction Ascending.

1. In 26242 farthings, how many pounds, shillings, and pence?

OPERATION.

| | | |
|---|---|---|
| 4 ) 26242 | | |
| 12 ) 6560 | . | 2far. rem |
| 2\|0 ) 54\|6 | . | 8d. rem. |
| 27 | . . | 6s. rem. |

*Ans.* £27 6s. 8d. 2far.

ANALYSIS.—Since 4 farthings make 1 penny, we first divide by 4. Since 12 pence make 1 shilling, we next divide by 12. Since 20 shillings make 1 pound, we next divide by 20, and find that 26242 farthings = £27 6s. 8d. 2far.: Hence, the following

# Addition of Denominate Numbers

**Fish, 1883**

**2 pints (pt) = 1 quart (q)**

**8 quarts (q) = 1 peck (pk)**

**4 pecks (pk) = 1 bushel (bu)**

## WRITTEN EXERCISES.

1. What is the sum of 32 bu. 2 pk. 6 qt.; 24 bu. 1 pk. 4 qt.; 16 bu. 3 pk. 7 qt.?

EXPLANATION.—Write the numbers so that units of the same denomination stand in the same column, and begin at the right to add.

| bu. | pk. | qt. |
|---|---|---|
| 32 | 2 | 6 |
| 24 | 1 | 4 |
| 16 | 3 | 7 |
| 74 | 0 | 1 |

The sum of the quarts is 17 qt., equal to 2 pk. 1 qt. Write the 1 qt. under the column of quarts, and add the 2 pk. to the column of pecks.

Add, in like manner, the column of pecks and bushels.

2. What is the sum of $\frac{7}{10}$ wk., $\frac{3}{5}$ da., and $\frac{3}{8}$ hr.?

# Subtraction of Denominate Numbers

**Fish, 1883**

24 grains (gr) = 1 pennyweight (pwt)

20 pwt = 1 ounce (oz)

12 oz = 1 pound (lb)

**202. To find the difference between any two denominate numbers, or denominate fractions.**

1. From 16 lb. 8 oz. 6 pwt. 10 gr., take 7 lb. 4 oz. 12 pwt. 6 gr.

| lb. | oz. | pwt. | gr. |
|---|---|---|---|
| 16 | 8 | 6 | 10 |
| 7 | 4 | 12 | 6 |
| 9 | 3 | 14 | 4 |

EXPLANATION.—Write the numbers as in addition.

Subtract 6 gr. from 10 gr., and write the difference, 4 gr., under the grains.

Since 12 pwt. cannot be subtracted from 6 pwt., take 1 oz., equal to 20 pwt., from the 8 oz., leaving 7 oz., and add it to the 6 pwt., making 26 pwt.; 12 pwt. from 26 pwt. leaves 14 pwt., which write under the pennyweights.

Since 1 oz. was taken from 8 oz., subtract 4 oz. from 7 oz., and write the difference, 3 oz., under the ounces. 7 lb. from 16 lb. leaves 9 lb., which write under the pounds. Hence, etc.

# Multiplication of Denominate Numbers

**Ray, 1880**

MULTIPLICATION OF COMPOUND NUMBERS.

**223. Compound Multiplication** is the process of multiplying a Compound Number by an Abstract Number.

PROBLEM.—Multiply 9 hr. 14 min. 8.17 sec. by 10.

OPERATION.

| da. | hr. | min. | sec. |
|---|---|---|---|
| | 9 | 14 | 8.17 |
| | | | 10 |
| 3 | 20 | 21 | 21.7 |

SOLUTION.—Ten times 8.17 sec. = 81.7 sec. = 1 min. 21.7 sec. Write 21.7 sec. and carry 1 min. to the 140 min. obtained by the next multiplication. This gives 141 min. = 2 hr. 21 min. Write 21 min. and carry 2 hr. This gives 92 hr. = 3 da. 20 hr.

# Multiplication of Denominate Numbers

Thomson, 1851

4 farthings(qr or far) = 1 penny (d)

12 pence (d) = 1 shilling (s)

20 shillings (s) = 1 pound (£)

2. What will 28 horses cost, at £21, 3s. $7\frac{1}{4}$d. apiece?

*Operation.*

| £ | *s.* | *d.* | *far.* | |
|---|---|---|---|---|
| 21 | 3 | 7 | 1 | |
| | | | 7 | |
| 148 | 5 | 2 | 3 | |
| | | | 4 | |
| 593 | 0 | 11 | 0 | *Ans.* |

We multiply by the factors of 28, which are 7 and 4, and set down each result as above.

# Multiplication of Denominate Numbers

<u>Maglathlin</u> & <u>Greenleaf</u>, 1831

Denominations of dry measure.

2 pints (pt) = 1 quart (q)

8 quarts (q) = 1 peck (pk)

4 pecks (pk) = 1 bushel (bu)

Dry measures are used in measuring grain, roots, fruits, etc.

## MULTIPLICATION.

**151.** Multiply 11 bu. 3 pk. 2 qt. by 7.

| | | | |
|---|---|---|---|
| 11 bu. | 3 pk. | 2 qt. | |
| | | 7 | |
| 82 bu. | 2 pk. | 6 qt. | |

*Solution.* — 7 times 2 qt. are 14 qt. = 1 pk. and 6 qt. We write the 6 qt. under the quarts, and reserve the 1 pk. to add to the 7 times 3 pk.

7 times 3 pk. = 21 pk., which plus the pk. reserved = 22 pk. = 5 bu. and 2 pk. We write the 2 pk. nder the pecks, and reserve the 5 bu. to add to the 7 times 11 bu.

7 times 11 bu. = 77 bu., which plus the 5 bu. reserved = 82 bu.

.ns. 82 bu. 2 pk. 6 qt.

# Division of Denominate Numbers

Ray, 1880.

DIVISION OF COMPOUND NUMBERS.

224. **Division of Compound Numbers** is the process of dividing when the dividend is a Compound Number. The divisor may be Simple or Compound, hence there are two cases:

1. *To divide a Compound Number into a number of equal parts.*
2. *To divide one Compound Number by another of the same kind.*

NOTE.—Problems under the second case are solved by reducing both Compound Numbers to the same denomination, and then dividing as in simple division.

# Division of Denominate Numbers

Ray, 1880

Denominations of Commercial weight

16 ounces (oz) = 1 pound (lb)

25 pounds (lb) = 1 quarter (qr)

4 quarters (qr) = 1 hundred-weight (cwt)

20 cwt = 1 ton (T)

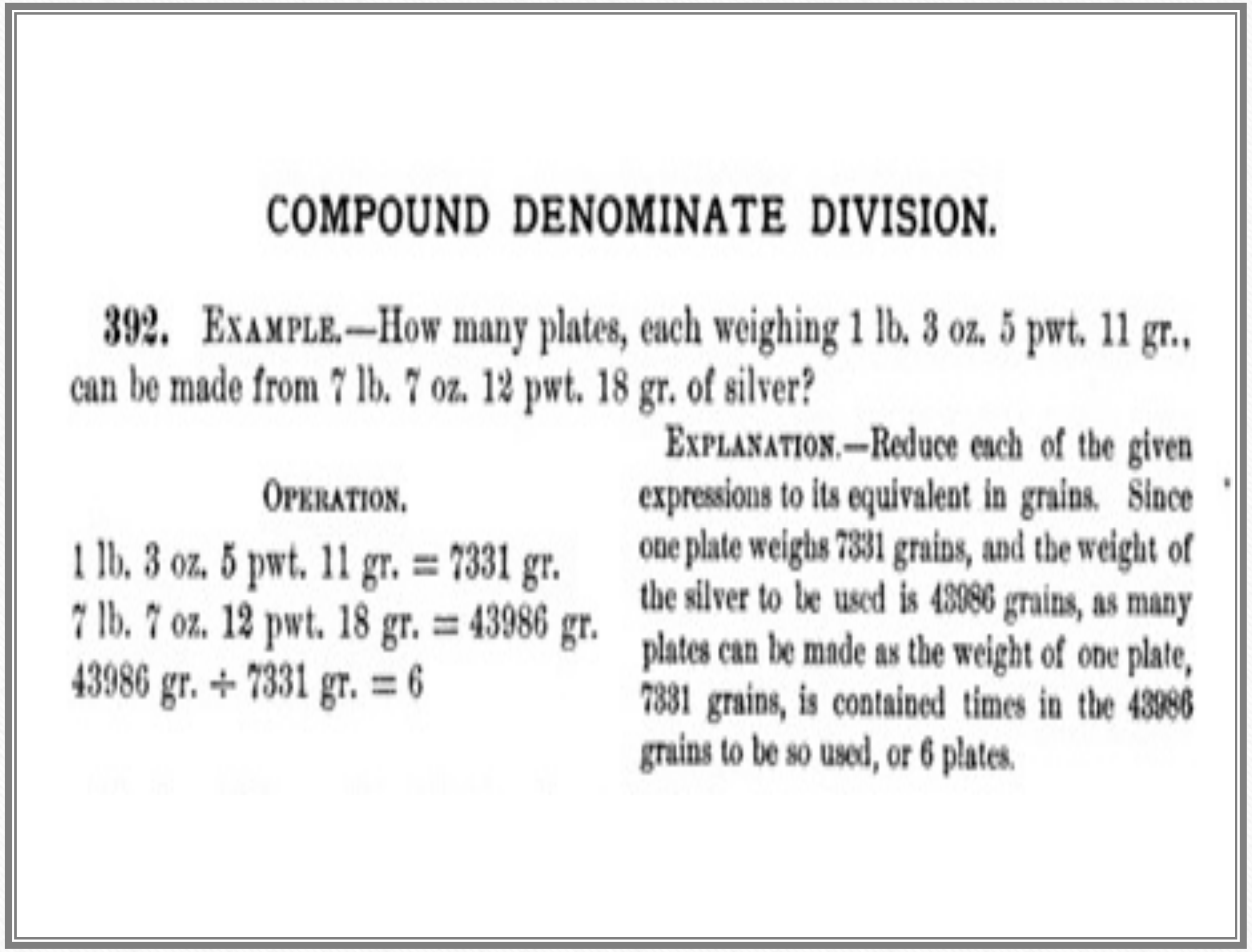
PROBLEM.—Divide 5 cwt. 3 qr. 24 lb. $14\frac{3}{4}$ oz. of sugar equally among 4 men.

SOLUTION.—4 into 5 cwt. gives a quotient 1 cwt., with a remainder 1 cwt., = 4 qr., to be carried to 3 qr., making 7 qr.; 4 into 7 qr. gives 1 qr., with 3 qr., = 75 lb., to be carried to 24 lb., = 99 lb.; 4 into 99 lb. gives 24 lb., with 3 lb., = 48 oz., to be carried to $14\frac{3}{4}$ oz., making $62\frac{3}{4}$ oz.; 4 into $62\frac{3}{4}$ oz. gives $15\frac{11}{16}$ oz., and the operation is complete.

OPERATION.

| | cwt. | qr. | lb. | oz. |
|---|---|---|---|---|
| 4) | 5 | 3 | 24 | $14\frac{3}{4}$ |
| | 1 | 1 | 24 | $15\frac{11}{16}$ |

# Division of Denominate Numbers



## COMPOUND DENOMINATE DIVISION.

**392.** EXAMPLE.—How many plates, each weighing 1 lb. 3 oz. 5 pwt. 11 gr., can be made from 7 lb. 7 oz. 12 pwt. 18 gr. of silver?

OPERATION.

1 lb. 3 oz. 5 pwt. 11 gr. = 7331 gr.
7 lb. 7 oz. 12 pwt. 18 gr. = 43986 gr.
43986 gr. ÷ 7331 gr. = 6

EXPLANATION.—Reduce each of the given expressions to its equivalent in grains. Since one plate weighs 7331 grains, and the weight of the silver to be used is 43986 grains, as many plates can be made as the weight of one plate, 7331 grains, is contained times in the 43986 grains to be so used, or 6 plates.

## No author given, 1897

24 grains(gr) = 1 pennyweight (pwt)

20 pwt = 1 ounce (oz)

12 oz = 1 pound(lb)

Descending : 12,20,24

Ascending: 24,20,12

# First Observation

The books treat only the following:

- Multiplication of denominate numbers by abstract numbers
- Division of denominate numbers into equal parts
- Division of denominate numbers by another of the same denomination

Rules are given to perform these operations.

In arithmetic operations involving reduction of denominate numbers, carrying and borrowing depends on the ratio of the units.

So each operation is performed differently if the ratio is different.

Thus when $19^{th}$ century students learned about units, they had to learn arithmetic that was quite different from what students learn today.

In the United States, decimals were not universally used in the 19th century, and this created a stress on the practical arithmetic that was taught in schools.

For example, in these textbooks we see pounds, shillings, pence, and farthings, remnants of the colonialist past of the United States.

The National Coinage Act of 1792 introduced decimal money, but it was slow to show up in arithmetic books.

And other English measures (weights, cloth, etc.) continued in arithmetic for many years.

- School Arithmetics Grammar School Book,Florian Cajori, 1915.



(5) time or duration, (6) temperature, (7) value or money.

Other measures which are rapidly coming into common use are those applied to measurement of energy or power, as electricity and light. Such measures as volts, amperes, and watt-hours are common. The meter registers in watt-hours, for which there is a charge of about 8 cents per watt-hour. Incandescent lamps are marked 16 or 32 candle power, or 40 or 60 watts, and so on. The power of an engine is estimated in horse-power as 3 h.p., or 60 h.p.

There are other systems of measure which may be regarded as defined, used in counting certain materials as units, dozen, gross, of anything, as brooms, eggs, pencils, pens, etc.; and in the paper trade one meets such terms as sheets, quire, ream, each of which has a definite denotation.

There are two systems of measurement in wide use. In the United States and Great Britain the English system is employed, while in the majority of the other nations the metric system is commonly employed. In scientific work everywhere the metric measures are necessary.

# Second Observation

- Scientific measures were not discussed as part of denominate numbers in the books I studied.
- Florian Cajori is the only author who made mention of some scientific measures.
- There were no units of rate like miles per hour, feet per second, mass per volume.
- There were no scientific measures, e.g., velocity, acceleration, force, etc.
- What we now call dimensional analysis was not a part of arithmetic books in the 19th century.

# Conclusion

Basic arithmetic in the books I studied was very complex because the arithmetic algorithms were different depending on which denominations they contained.  This complexity was decreased when decimal arithmetic started being more dominant.

Of course the 20th century brought many other changes to arithmetic, for example, abstraction. And techniques of dealing with denominate numbers such reduction ascending and descending were dropped.