Teaching Fractions Using Counting Boards

TSG 8: Teaching and learning of arithmetic and number systems
(focus on primary education)
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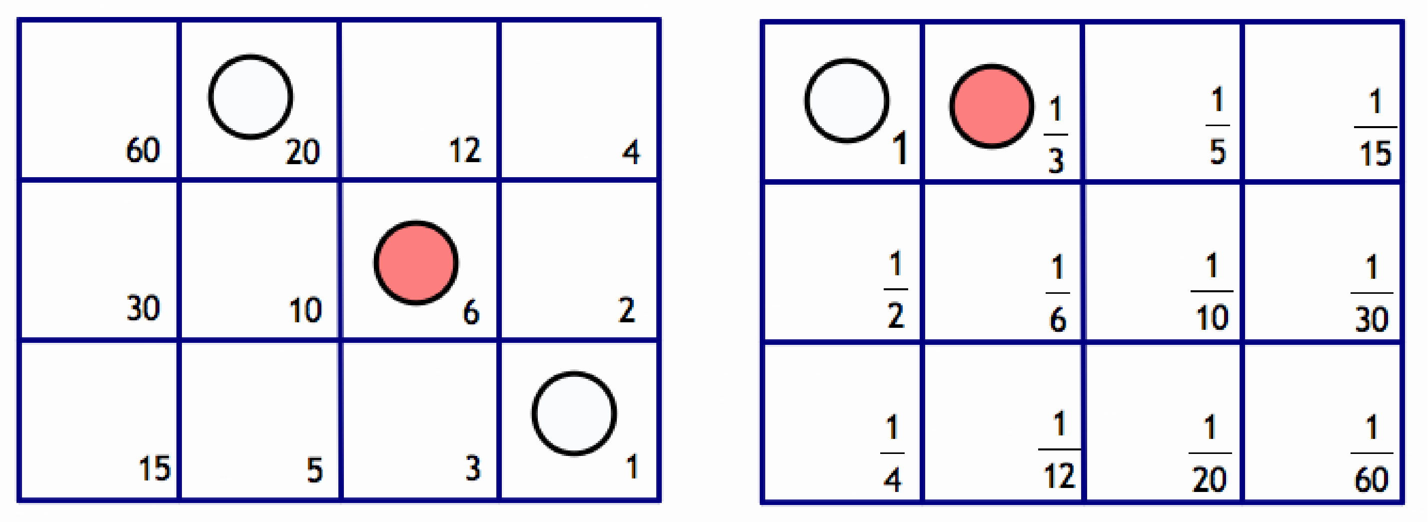
Introduction

We have been experimenting with using counting boards of different sizes as a platform for teaching the arithmetic of rational numbers in elementary and middle grades.



The values of locations in any column *always* form a geometric progression with quotient 2.
Tokens are of two colors. White tokens have the value 1 and red tokens have the value -1. So the three tokens on the first board represent the number 15 = 20 + -6 + 1, and the two tokens on the second board represent the value 2/3 = 1 – 1/3.

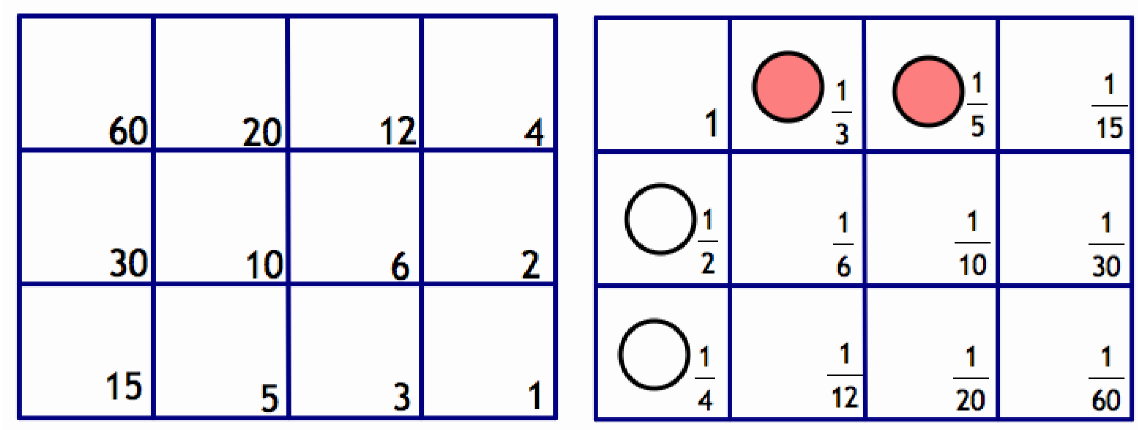
Representing numbers on boards, instead of writing them down, has the following advantages.

1. Students can carry out fairly complex computations before they master “number facts”.

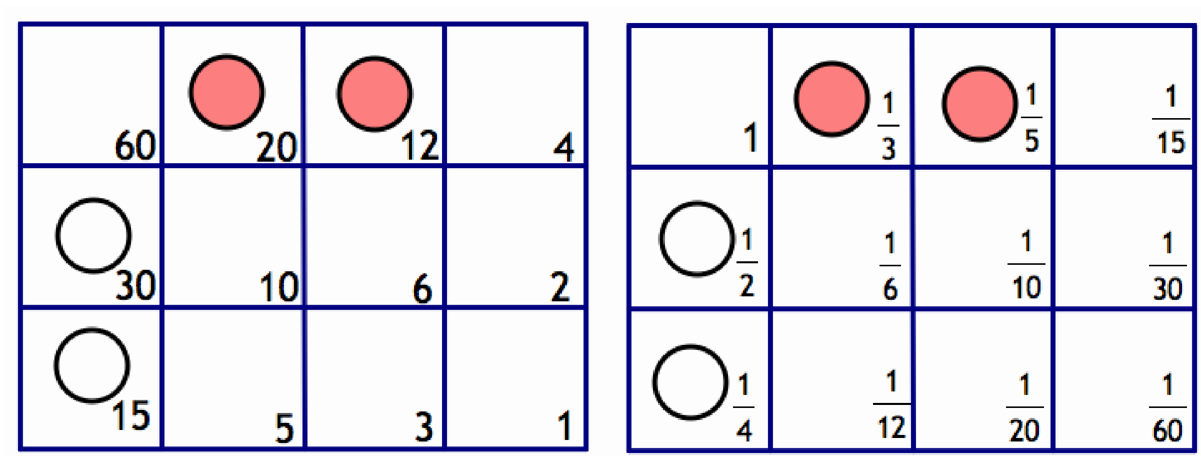
2. Good penmanship is not required.

3. Only very limited knowledge of number-words is necessary. And this feature is important in multi-lingual classrooms.

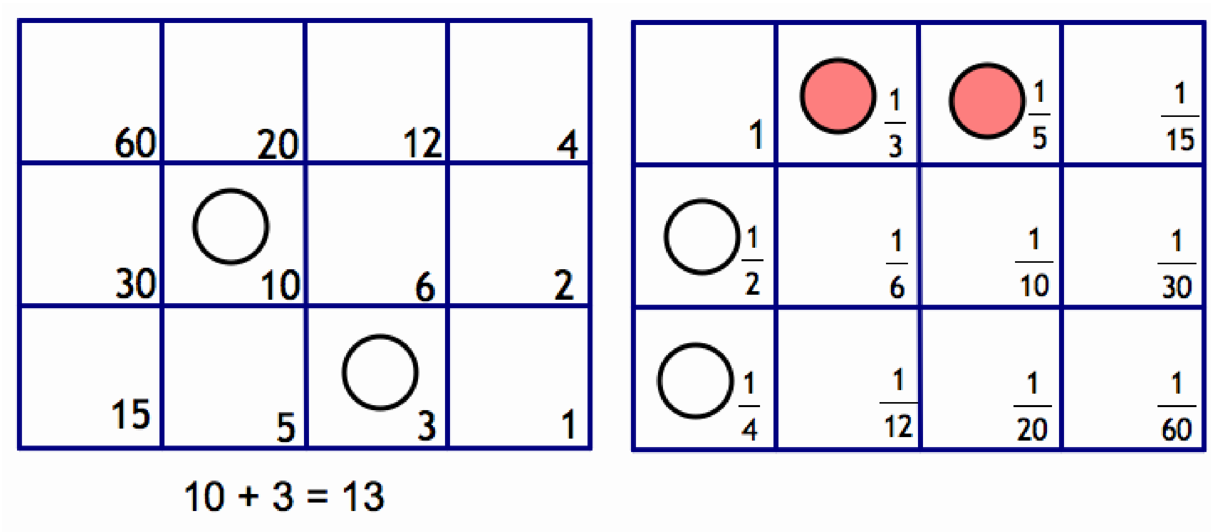
Addition and subtraction of fractions whose denominators are factors of 60



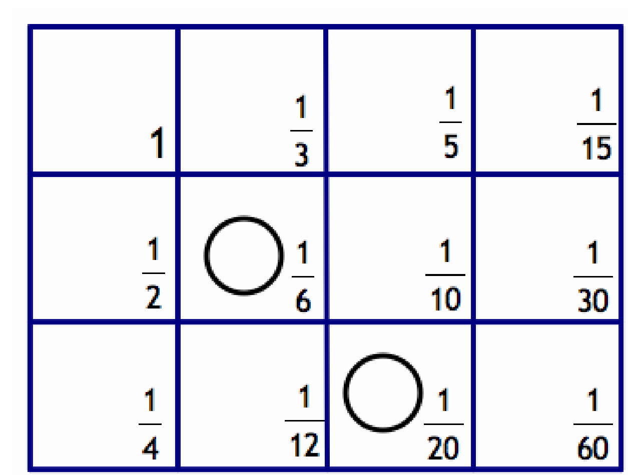
On the fraction board we have a number, ½ - 1/3 + ¼ - 1/5.
To compute the total we put the same configuration of tokens on the integer board.



Now we regroup these tokens, getting 30 - 20 + 15 – 12 = 10 + 3 = 13.



We copy this onto the fraction board:



This configuration on the fraction board means that

½ - 1/3 + ¼ - 1/5 = 1/6 + 1/20 = 13/60

This method of computation is justified by distributivity of multiplication. The values on the integer board are the values from fraction board multiplied by 60.

The size of the board that is used depends on the range of dominators we want to handle.
Computing with denominators that contain many prime factors requires fairly large boards and more advanced techniques.

Other examples

The boards have been used for three different purposes:

(1) To teach standard algorithms for addition, subtraction, multiplication, and division

(2) To do hands-on tasks whose completion requires complex arithmetic computations, without using calculators or computers

(3) To investigate properties of rational numbers

Very few students (preservice teachers and elementary/middle school pupils) have shown interested in activities involving standard algorithms.

Hands-on activities were interesting for most students from all backgrounds.

Even college students seem to prefer mathematical problems that arise from meaningful physical activities.

Investigations of properties of rational numbers have been the most popular. They are not time consuming; they are interesting to most students, and often produce surprising and unexpected results.

An example
In how many ways can you represent the number 3 on a specific board using only two tokens? (You may use negative numbers and fractions.)

60	20	12	4
30	10	6	2
15	5	3	1

1	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$
$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{30}$
$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{20}$	$\frac{1}{60}$

Adding fractions, as we have shown above, is a rather advanced topic in elementary and middle grades.

But the same two boards have also been used with young learners for telling time.

When they put the number of minutes on the integer board, the same configuration of tokens on the fraction board shows the same length of time as a fraction of an hour.

Curricular materials

B r eaking Away from the Mathbook

by
Pat Baggett and Andrzej Ehrenfeucht

Creative Projects for K-12
[en Español]

eBook of Calculus Projects

Number Boards

Arithmetic Algorithms

Statistics

Breakingaway | Notes | Credits | Album | NMSU Courses for Teachers | Contact Us

Maintaned and updated by Chant Gant , original design by Aous Manshad | last modified : August 11, 2013

This educational resource website has received 000214259 visitors since Feb 6, 1999!
web counter

https://www.math.nmsu.edu/~breakingaway/

Number Boards

Index of Lessons

- Number Boards: An Introduction
- Ways to make 0 to 12 on the 2x2 board
- Parts of a Circle Using Sectors and Counting Boards
- Fractions with Clay and Counting Boards
- Adding and Subtracting Fractions

https://www.math.nmsu.edu/~breakingaway/NumberBoards/index.html

Final comments

References

All are on our website https://www.math.nmsu.edu/~breakingaway/
under Notes, https://www.math.nmsu.edu/~breakingaway/notes/notes.html

(1) Counting, JMM, New Orleans, LA January 6, 2011
(2) Arithmetic Algorithms Taught in Schools, JMM, Baltimore, MD January 15-18, 2014
(3) Mathematical Properties of Decimal Counting Boards, JMM, San Antonio, TX, January 11, 2015
(4) Explaining Arithmetic Algorithms, 54th Annual Meeting, Psychonomic Society, Toronto, Ontario, Canada, November 14-17, 2013
(5) Two Models of Learning the Concept of Whole Numbers, Annual Meeting, Psychonomic Society, Long Beach, CA, November 20-23, 2014
History of mathematic education:
(6) Number Words and Arithmetic, HPM-American Section Meeting, University of California, Berkeley, October 26-28, 2012
(7) Was John Napier the First Modern Computer Scientist? Joint Meeting, HPM and The Midwest History of mathematics Conference, Wabash College, Crawfordsville, IN, October 18, 2014.
(8) A Prehistory of Arithmetic, MathFest (History and Philosophy of Mathematics), Washington DC, August 8, 2015

Thank you