

# Algebra in Experimental Courses for Teachers

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### INTRODUCTION

Algebra had its beginnings as a technique for solving equations. After Newton, it became the main tool for creating models of real events, "laws of nature". During the twentieth century, algebraic formulas spread to all domains of mathematics, and in the emerging domain of computer science formulas have played the role of basic sentences in programming languages. Methods of solving equations form only a very small part of modern algebra; but they are still the main focus of the algebra that is taught in schools today.

### Courses for current and future teachers at New Mexico State University

In Las Cruces, NM, USA, we offer a group of courses in which algebra is taught through a sequence of concrete tasks. Students are presented with a task involving a physical activity such as drawing plans, making objects, and measuring amounts. In order to complete the given task successfully they have to construct a mathematical model of the situation, use appropriate formulas, perform needed computations using programmable calculators, and finish the physical part of the work.

### Some differences between traditional courses and the courses we offer

TRADITIONAL ALGEBRA COURSES	OUR COURSES
Variables are symbols to be replaced by numbers	Variables always have meaning
Most formulas that students see describe relationships among abstract numbers	Most formulas that students see represent relationships among physical quantities
Manipulating formulas is the main activity	Formulas are transformed only when required by the context
Real numbers are avoided, and computations are exact	Real numbers are the default on all levels; computations are done with calculators
Algebra is a topic by itself	Algebra is always used as part of a broader investigation

### Students' comments and evaluations of the courses

- "The materials were extremely useful and I will implement many of these lessons into my future classrooms."
- "[The materials] allow me a huge variety of ways to solve and complete problems while enjoying it."
- "I found all the materials interesting and fun. The format was really well thought out."
- "Many lessons can be adapted to different grade levels."
- "It was hard. I spent hours with this course, but it has helped me the most."
- "I am definitely saving my binder of lessons and journals."
- "The class helped me get fresh ideas that kids will enjoy. ... I love them all... Teach the same ones next year!"
- "I think the hands-on material was very beneficial. It allowed us to see visually the problems."
- "This is a very well rounded class and everything we learned was useful."

### Materials and articles about the courses

Breaking Away website with over 100 lesson plans, elementary through calculus  
<http://sofia.nmsu.edu/~breakingaway/>

#### BOOKS OF LESSON PLANS

Baggett, P. & Ehrenfeucht, A. (2001). *Breaking away from the algebra and geometry book: Original and traditional lessons for grades K-8*. Lanham, MD: Rowman & Littlefield.

Baggett, P. & Ehrenfeucht, A. (2004). *Breaking away from the math book: Creative projects for grades K-6*. Lanham, MD: Rowman & Littlefield.

Baggett, P. & Ehrenfeucht, A. (2004). *Breaking away from the math book II: More creative projects for grades K-8*. Lanham, MD: Rowman & Littlefield.

Baggett, P. & Ehrenfeucht, A. (2003). *Breaking away from the math and science book: Physics and other projects for grades 3-12*. Lanham, MD: Rowman & Littlefield.

#### ARTICLES

Baggett, P. & Ehrenfeucht, A. (2008). Coordinating abstract, physical, and computational aspects of algebra in a single lesson. <http://sofia.nmsu.edu/~breakingaway/works/AAA/AAA.html>

Baggett, P. & Ehrenfeucht, A. (2011). Negative numbers in school mathematics. [http://sofia.nmsu.edu/~breakingaway/notes/Lexington\\_MathFest.pdf](http://sofia.nmsu.edu/~breakingaway/notes/Lexington_MathFest.pdf)

## EXAMPLES OF TASKS

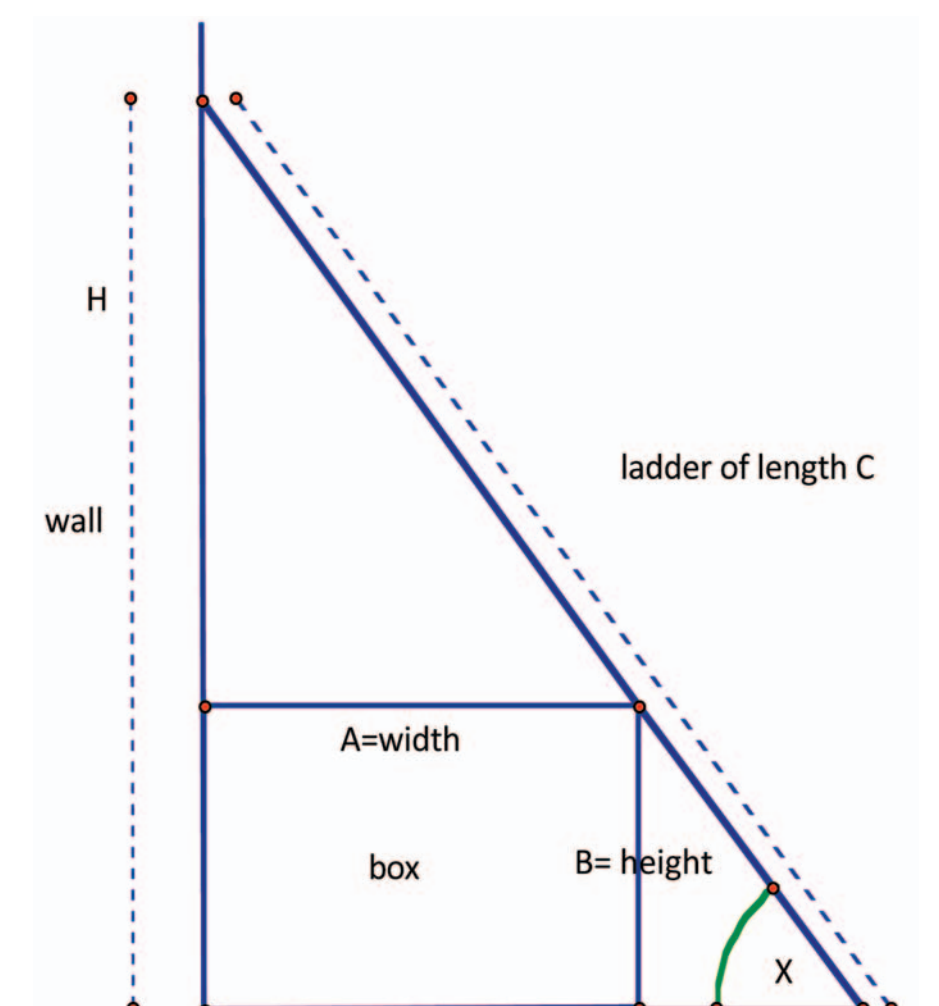
### 1. LADDER AND BOX PROBLEM



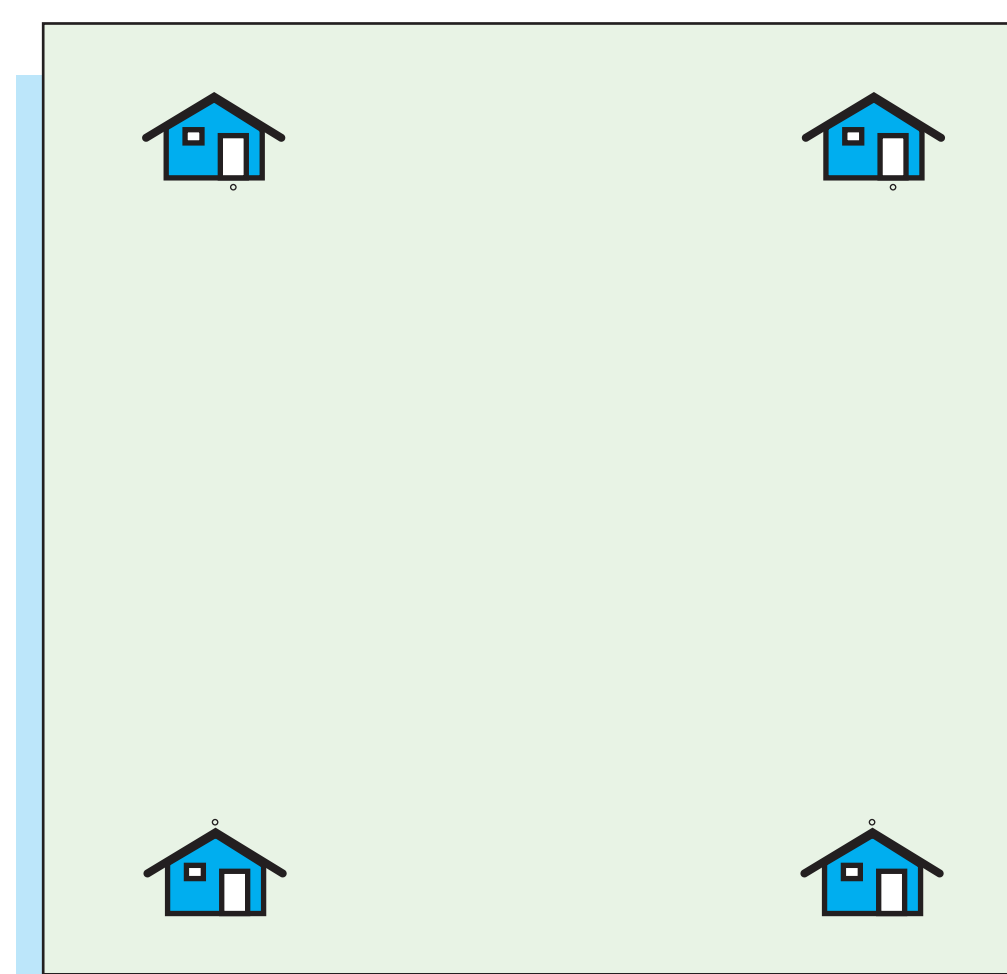
Each group of students is given a box or a wooden block, a stick or ruler representing a ladder, and a scientific calculator, TI-84. They put the ladder next to the wall over the box as shown. At what height  $H$  does the ladder touch the wall?

The task is:

- Measure the width and  $A$  and length  $B$  of the box, and the length  $C$  of the stick, and compute  $H$ .
- Find a general way of solving this problem for any values  $A$ ,  $B$ , and  $C$ .
- Put the box next to a wall or any other vertical surface, put the stick next to it and measure how high up it touches the wall.
- Compare the predicted and actual results, and discuss the whole problem.



### 2. SHORTEST ROAD



#### QUESTION

What is the shortest road that can be constructed to connect four houses at the corners of a square one mile on a side? Show how you solved this problem, and draw a picture with your solution.

### 3. BANANAS



#### QUESTIONS

What percentage of the weight of a banana is the edible part?  
 What percentage of the weight is the peel?

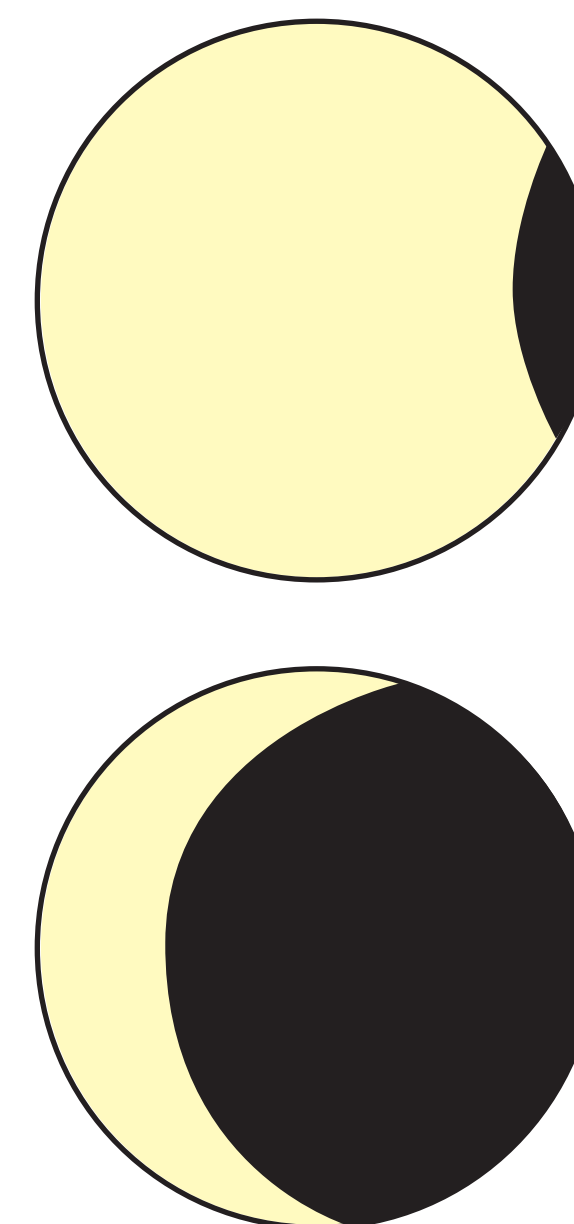
This problem is for children in early grades. Each group needs a whole banana (with peel), scales, plastic ziplock bag, and calculator. They first estimate the percentage of the weight of the banana's peel. Then they weigh the whole banana, and weigh separately its peel and its edible part. They record the results and use a four-operation calculator to perform the arithmetic operations.

#### EXPECTED ANSWER:

(The weight of the peel is about 35% of the weight of the banana.)



### 4. ECLIPSE OF THE SUN



The sun and the moon, viewed from earth, appear to be approximately the same size.

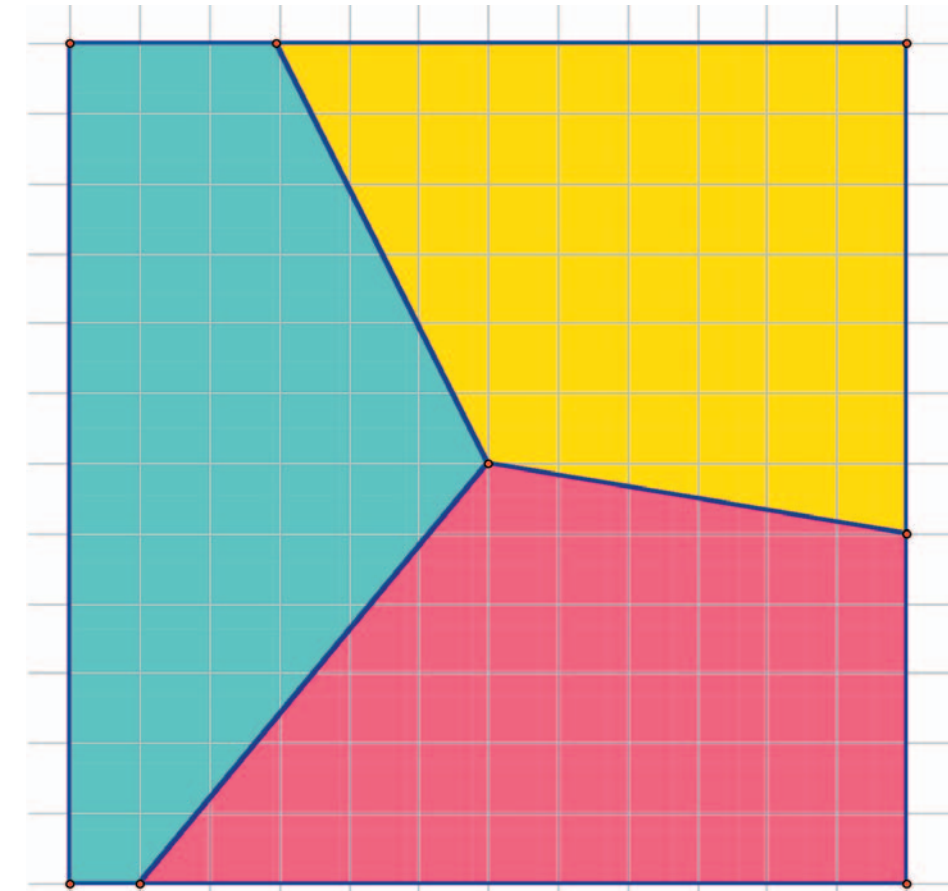
#### TWO QUESTIONS:

- Given a picture of a partial eclipse of the sun, what percentage of the sun is covered?
- Draw a picture in which exactly one half of the sun is covered by the moon, and prove that your answer is correct.

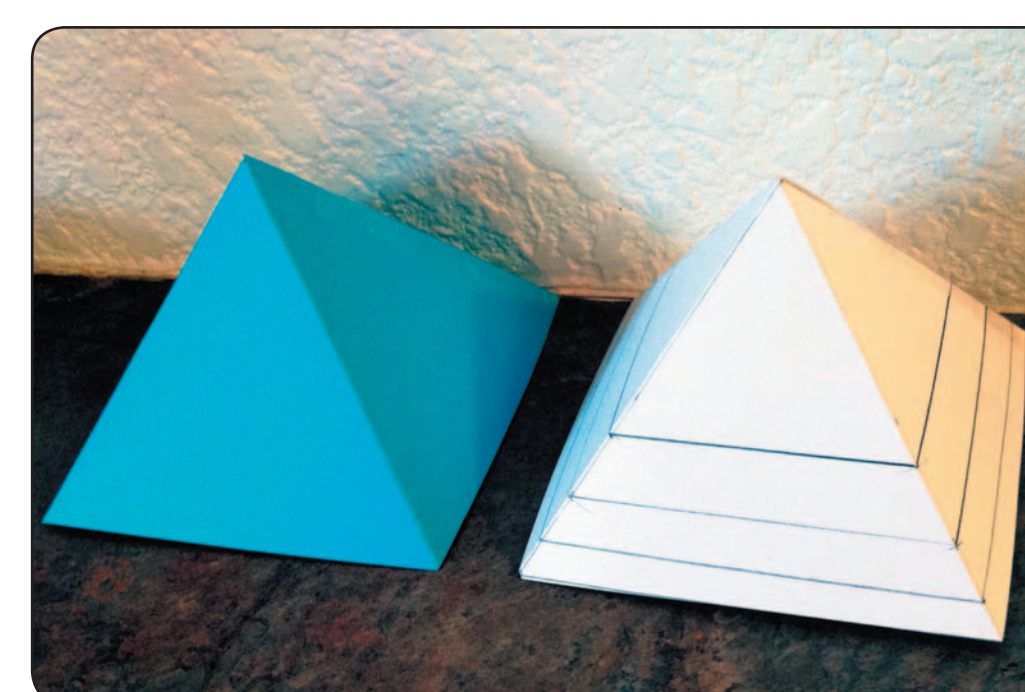
### 5. SQUARE PIZZA

A new pizza place in town serves only square pizzas, and it also makes the best crust. Ann, Bruce, and Carlos want to share a pizza, with each getting one piece, and all three having the same amount of pizza and the same amount of crust. (Here we assume the crust is the same as the outer perimeter.) You have a square. How to divide the pizza?

(One solution is shown, but there are infinitely many ways to do it!)



### 6. PYRAMID INTO FOURTHS



Construct from poster board a pyramid with a square base having sides 10 cm long that has equilateral triangles for faces. On each face draw three horizontal lines marking three different levels, for 1/4 of the volume, 1/2 of the volume, and 3/4 of the volume of the pyramid. (The cuts are at 6.3, 7.9, and 9.1 cm along the edge from the apex (see drawing), a fact that is surprising for most students. This can be checked using rice.)

### 7. NEGATIVE NUMBERS IN EARLY GRADES

Children record the temperature outside their classroom at noon every day and keep a chart of the temperatures. They also record the difference between the temperatures on two consecutive days. The use of negative and positive numbers is explained as follows.

When we record changes we use positive and negative numbers. Positive numbers show that the temperature increased, and negative numbers show that it decreased.

