A story problem about a privateer in 19th century American calculus books

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Outline

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1. Background: Early calculus books published in the United States

Florian Cajori (*The Teaching and History of mathematics in the United States, 1890*) gives a list in chronological order of calculus books published in the United States.
The first book in his list is Charles Hutton's 1812 *A Course of Mathematics (vol. 2)*.
He lists only ten others published in the United States by 1851.
The twelfth book in his list is by William Smyth (1854), *Elements of the Differential and Integral Calculus*.

And this is when our story begins.

2. Privateers and the privateer problem

Beginning in 1854, and lasting until 1922, a “story” problem about a privateer appeared in American calculus books by six authors.

Below we examine renditions of the problem.

But first a little history about privateers.
(We thank Rob Ossian, http://www.thepirateking.com/terminology/definition_privateer.htm, for much of this information.)

a. What is a privateer?

“Privateer” in international law is the term applied to a privately owned armed vessel whose owners are commissioned by a hostile nation to carry on naval warfare.

Such naval commissions or authorizations are called “letters of marque”.

Privateering is distinguished from piracy, which is carried out without enlistment of a government.

b. History and laws regarding privateers

Privateering was abolished by the Declaration of Paris of 1856, but the declaration was not supported by the United States, Spain, Mexico, and Venezuela.

The Hague Conference of 1907 prescribed the conditions under which a private merchant vessel converted to war purposes has the status of a warship.

Under the U.S. Constitution, Congress has the power to issue "letters of marque" and therefore to make use of privateers, to this day.

The practice of privateering preceded the creation of national navies.

During the Middle Ages, European states having few or no warships hired merchant vessels for hostile purposes.

The issuing of “letters of marque” to ship owners or procurers, authorizing them to prey on the commerce of the enemy, eventually came into general use.

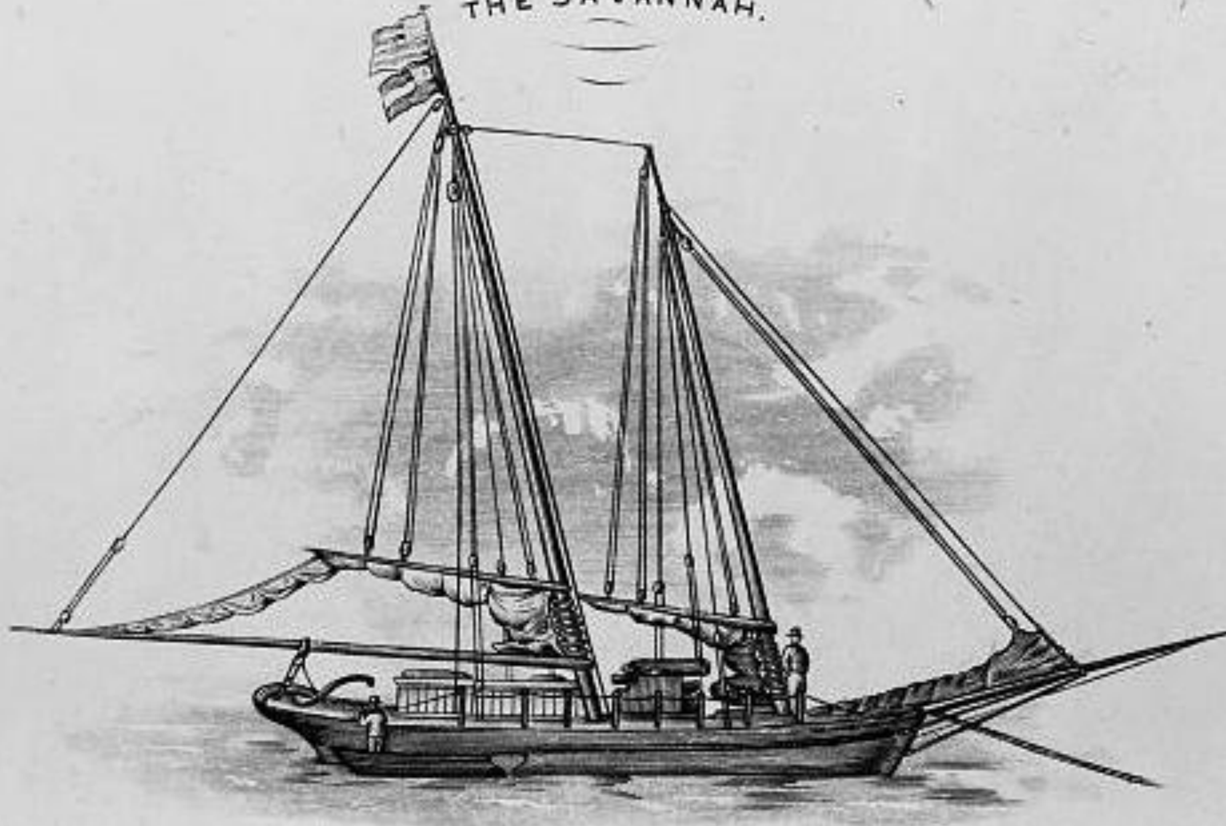
By way of compensation, privateers were allowed to share any booty captured.

Privateering was carried on during the American Revolution and the War of 1812.

c. Privateering during the Civil War

Congress authorized President Lincoln to commission privateering in 1863 during the Civil War, but the power was not exercised; the Confederacy, however, engaged in privateering during this period.

THE SAVANNAH.



THE CONFEDERATE STATES PRIVATEER SAVANNAH, LETTER OF MARQUE Nº 1, CAPTURED OFF CHARLESTON, BY THE U.S. BRIG FERRY, LIEUT. PARROTT.

Entered according to act of Congress in the Year 1861 by E. E. KIMMEL 59 NASSAU St N.Y. in the Clerks Office of the District Court for the Southern District of New York.

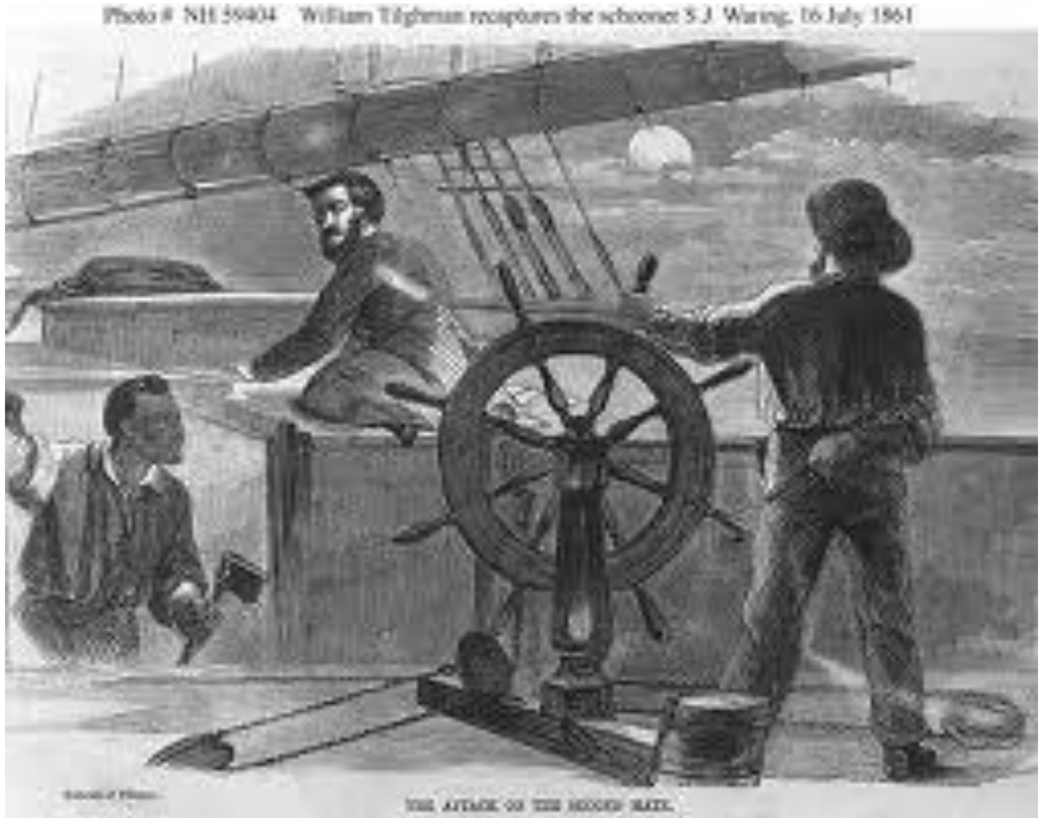
Privateering was expressly renounced by the United States during the Spanish-American War of 1898.

d. An article about a privateer in the Civil War

Civil War at Sea from S*cientific American*, August 2011, p. 94. 150 years ago

"The schooner *S.J. Waring*, which had been captured by the privateer Jeff. Davis, arrived in this port on Sunday, July 21st [1861], having been retaken by the black steward, with the assistance of one of the seamen. When the *S.J. Waring* was taken, her captain and mate were taken off, but the colored steward, two of the seaman and a passenger were left on board. The steward having discovered, by a convention which he heard, that it was the intention of the prize master Capt. Amiel to sell him into slavery as soon as the schooner arrived in Charleston, determined to make a desperate attempt to retake the vessel. The steward's name is William Tillman."

An account of this event is at www.ScientificAmerican.com/aug2011/waring



Depiction of William Tillman, a merchant ship’s cook who, as written by some historians, took up arms (a hatchet) to prevent being sold into slavery after a Confederate raider captured his vessel.

e. First appearance of the privateer problem

There was a resurgence of privateering in the Confederacy during the Civil War.
And as mentioned, the "privateer problem" appears first in a calculus book, *Elements of the differential & integral calculus*, by William Smyth, in 1854.
(There are two other problems about privateers that occur in arithmetic and algebra textbooks: sharing a booty (arithmetic), and path of pursuit (trigonometry), but we will not discuss them.)

The calculus problem is about a privateer attempting to sail unnoticed between two lights on opposite shores.
From Smyth:

9. A privateer wishes to get to sea unobserved, but has to pass between two lights A and B, on opposite head lands. The intensity of the lights is known and the distance between them. At what point between them must she pass so as to be as little in the light as possible.

Let $a =$ the given distance AB; $b =$ the intensity of the light A at the unit of distance, $c =$ the intensity of B at the same distance. Let $x =$ the distance of the point sought from A. Then since, by the principles of Optics, the intensities are inversely as the squares of the distances, we shall have for the illumination or intensity of light upon the point sought

$$\frac{b}{x^2} + \frac{c}{(a - x)^2},$$

which by the question should be a minimum.

Ans. $x = \dfrac{ab^{\frac{1}{3}}}{b^{\frac{1}{3}} + c^{\frac{1}{3}}}$.

A mathematical problem regarding the smallest illumination was, and is, very common in calculus texts. And it is usually presented without involving any story. For example, in *Differential and Integral Calculus* by Richard Courant, 1934, p. 166, we see:

6. Two sources of light of intensities *a* and *b* are at a distance d apart. At which point of the line joining them is the illumination least? (Assume that the illumination is proportional to the intensity and inversely proportional to the square of the distance.)

3. Six authors of the problem

William Smyth (1797-1868)
Elements of the differential & integral calculus, 1854, p. 89 (Portland, ME)
Born in Pittston, ME; a professor of mathematics and natural philosophy at Bowdoin College in Brunswick, ME.
He wrote at least 6 widely used textbooks from algebra through calculus.

Edward Olney (1827-1887)
A general geometry and calculus, 1871, p. 100 (New York)
Born in Moreau, NY; a professor of mathematics at the Univ. of Michigan in Ann Arbor.
He wrote at least 13 books, from *The Science of Arithmetic* to geometry to algebra to calculus.

Edward Albert Bowser (1837-1910)
An elementary treatise on the differential and integral calculus, 1880, p. 170 (New York)
Born in New Brunswick, Canada; a professor of mathematics at Rutgers University, Newark, NJ.
He wrote at least one book per year over a decade, in mathematics and mechanics.

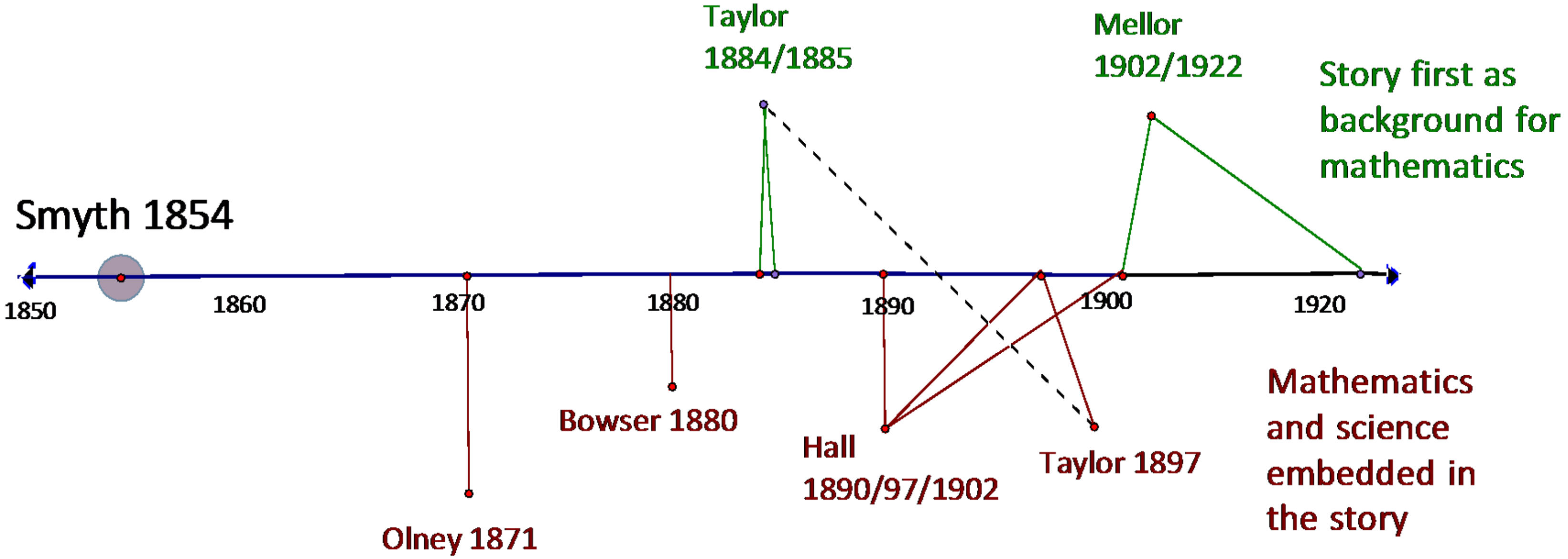
James Morford Taylor (1843-1930)
Elements of the differential and integral calculus with examples, 1884/85/97 p. 133 (Boston)
Born in Holmsdel, NY; a professor of mathematics at Colgate Univ, Hamilton, NY.
He wrote at least six books, on algebra, trigonometry, determinants, and calculus.

William Shaffer Hall (1873-1938)
Elements of the differential and integral calculus with applications, 1890/97/1902, p. 123 (New York)
Born in England; a professor of mathematics at Owens College in Manchester, England.
He wrote at least five books on geometry, mensuration, and calculus.

Joseph William Mellor (1869-1938)
Higher mathematics for students of chemistry and physics 1902/1922, p. 167 (London, NY, Bombay)
Born in England; a professor of chemistry, Staffordshire Univ.
He wrote at least 20 books on chemistry.

Timeline of the 7 appearances of the problem by the six authors in two different formats:

1. Story first as background for mathematics, and
2. Mathematics embedded in the story



Comparison and analysis of the two formats of the texts

We separately compare, line by line, the texts of the story of Smyth, Taylor (1884/5), and Mellor, because they keep the story and the math data separate.

And we separately compare Smyth (as the original source) with the texts of Olney, Bowser, Taylor (1897), and Hall, because they embed mathematical information into the story.

The first line, Smyth → Taylor (1884/5) → Mellor:

Smyth A privateer wishes to get to sea unobserved, but (A)
Taylor A privateer
Mellor A privateer

S has to pass between two lights A and B, on opposite head lands.
T has to pass between two lights A and B, on opposite headlands.
M has to pass between two lights A and B, on opposite headlands.

S The intensity of the lights is known and….the distance between them.
T The intensity of each light is known, and also the distance between them.
M The intensity of each light is known and also the distance between them.

S At what point between them must she pass (B)
T At what point must the privateer cross the line joining the lights
M At what point must the privateer cross the line joining the lights

S so as to be as little in the light as possible.
T so as to be in the light as little as possible?
M so as to be illuminated as little as possible?

The second line, Smyth → Olney → Bowser → Taylor (1897) → Hall:

Smyth A privateer wishes to get to sea unobserved, but
Olney A privateer wishes to get to sea unobserved, but
Bowser A privateer wishes to get to sea unmolested, but
(Taylor) A privateer
Hall A privateer

S has to pass between two lights A and B, on opposite head lands.
O has to pass between two lights A and B, on opposite headlands,
B has to pass between two lights, A and B, on opposite headlands,
(T) must pass between two lights A and B on opposite headlands,
H has to pass between two lights A and B on opposite headlands,

S …the distance between them [is known].
O the distance between which is a.
B the distance between which is c.
(T) the distance between which is c.
H the distance between which is c.

The second line, continued: Smyth → Olney → Bowser → Taylor (1897) → Hall

Smyth The intensity of the lights is known …
Olney The intensity, at a unit's distance, of A is b,
[N.B. "the intensity of A is b"!]
Bowser The intensity of the light A at a unit's distance is a,
(Taylor) The intensity of the light A at a unit's distance is a,
Hall The intensity of the light A at a unit's distance is a,

S …………
O and of B is c.
B and the intensity of ………… B at the same distance is b;
(T) and the intensity of light B at a unit's distance is b.
H and the intensity of …………… B at a unit's distance is b.

S At what point between them must she pass
O At what point must the privateer cross the line joining the lights
B At what point between the lights must the privateer pass
(T) At what point must a privateer pass the line joining the lights
H At what point must a privateer cross the line joining the lights

The second line, cont.: Smyth → Olney → Bowser → Taylor (1897) → Hall

Smyth so as to be as little in the light as possible.
Olney so as to be as little in the light as possible;
Bowser so as to be as little in the light as possible,
(Taylor) so as to be as little in the light as possible,
H so as to be as little in the light as possible,

S ……………………………………
O it being understood that
B assuming the principle of optics that
(T) assuming the principle of optics, that
H assuming the principle of optics, that

S …………………………………..
O the intensity of light at any point equals its intensity at a unit's
B the intensity of light at any distance equals its intensity at the
(T) the intensity of light at any point is equal to its intensity at a unit's
H the intensity of light at any point is equal to its intensity at a unit's

S ……………………………..
O distance divided by the square of the distance from the light.
B distance one divided by the square of the distance from the light.
(T) distance divided by the square of the distance of the point from the light.
H distance divided by the square of the distance of the point from the light.

4. Comparison & analysis of two formats of the text

The line Smyth → Taylor → Mellor represents one format in which a story is used in a mathematical problem.

In this approach, the story provides background information, which later has to be translated into mathematical jargon by declaring variables and providing formulas and mathematical and scientific facts.

The texts of these three, Smyth, Taylor (early), and Mellor (except changes in parts (A) and (B)) are almost identical:

Smyth A privateer wishes to get to sea unobserved, but (A)
Taylor A privateer
Mellor A privateer

S has to pass between two lights A and B, on opposite head lands.
T has to pass between two lights A and B, on opposite headlands.
M has to pass between two lights A and B, on opposite headlands.

…….

S At what point between them must she pass (B)
T At what point must the privateer cross the line joining the lights
M At what point must the privateer cross the line joining the lights

But the second (mathematical) parts of the problem (which we have not shown) are presented differently.

(In one of his later books (1897), Taylor used a different format for combining a story with a math problem, but then he returned to his first format.)

A second format

In the line Smyth→Olney→Bowser→Taylor (1897) →Hall, Olney changed the structure of the story.

Rather than separating the story and the mathematics, he embedded all mathematical and scientific information into the story:

Olney’s version (1871)

Ex. 19. A privateer wishes to get to sea unobserved, but has to pass between two lights, A and B, on opposite headlands, the distance between which is a. The intensity, at a unit's distance, of A is b, and of B, c. At what point must the privateer cross the line joining the lights, so as to be as little in the light as possible; it being understood that the intensity of a light at any point equals its intensity at a unit's distance divided by the square of the distance from the light.

SUG.—Letting $x =$ the distance from A, the function is $u = \frac{b}{x^2} + \frac{c}{(a - x)^2}$.

$$x = \frac{ab^{\frac{1}{3}}}{b^{\frac{1}{3}} + c^{\frac{1}{3}}}.$$

This is a different use of a story which merges completely with a mathematical problem.

But Olney preserved the details of Smyth's original version. The strange use of variable names, b for intensity of light A, and c for intensity of light B, were taken from Smyth, where they were described outside of the story itself. Bowser and Hall followed Olney's version of the problem, but they "corrected" his use of variables:

Smyth The intensity of the lights is known …
Olney The intensity, at a unit's distance, of A is b,
Bowser The intensity of the light A at a unit's distance is a,
(Taylor) The intensity of the light A at a unit's distance is a,
Hall The intensity of the light A at a unit's distance is a,

S …………..
O and of B is c.
B and the intensity of … B at the same distance is b;
(T) and the intensity of light B at a unit's distance is b.
H and the intensity of ………… B at a unit's distance is b.

But these two lines of dependencies,

(1) story first and math second (Smith→Taylor→Mellor), and
(2) story and math mixed together (Smith→ Olney→ Bowser→ Hall →Taylor),

do not describe all of the influences.

Taylor must have known Olney's version from the beginning, because he used a phrase introduced by Olney:

Olney At what point must a privateer cross the line joining the lights
Taylor At what point must a privateer cross the line joining the lights
(1884)

But he still chose to follow the plan of the original text of Smyth. The last sentences in Taylor's and Hall's 1897 problems are the same,

Taylor and Hall: assuming the principle of optics, that the intensity
(1897) of light at any point is equal to its intensity at a unit's distance divided by the square of the distance of the point from the light.

because they are part of the solution to the problem (but not part of the story) in Taylor's previous books.

In general, the texts of any two of the six authors differ in enough details for each author to claim that the problem was not copied, but just adapted from previous sources.

5. Why this problem?

We mentioned that stories about privateers (division of booty (arithmetic), and "chase" problems (trigonometry)) were present in nineteenth century math books.

But when Smyth used this theme in calculus, privateering, after the 1856 Paris Convention, seemed to belong to the past, and no one picked up the topic for 15 years.
But privateering recurred during the Civil War and, according to the article in *Scientific American* cited above, it achieved considerable notoriety.
So the original problem was revived by other authors, and it survived, mainly due to the popularity of Taylor's books, through the first quarter of the twentieth century.

6. Final remarks

In this talk we have traced the history of an individual problem about a privateer seeking safe passage, over a period of 68 years.

But we think that, in general, story problems in mathematics are an enigma. They are a persistent feature throughout all of history. They belong neither to theoretical nor to applied mathematics. And their pedagogical role is really unknown. Maybe a systematic study of the history of individual problems, which can be traced now with great ease due to online access to so many books, will give us better insight into how they propagate, and what the secret is of their longevity.

It is possible that story problems do not play any significant intellectual or pedagogical role, but that their persistence and propagation is an "accidental" byproduct of the way that mathematical knowledge spreads, when authors of mathematical texts "copy with modifications" the texts of their predecessors.

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James Morford Taylor (1884/85;1902, but not 1897), p. 133:

20. A privateer has to pass between two lights A and B, on opposite headlands. The intensity of each light is known, and also the distance between them. At what point must the privateer cross the line joining the lights, so as to be in the light as little as possible?

Let $d =$ the distance AB, and x the distance from A of any point P on AB. Let a and b be the intensities of the lights A and B respectively, at a unit's distance. By a principle of Optics, the intensity of a light at any point equals its intensity at a unit's distance divided by the square of the distance of the point from the light.

Hence $\frac{a}{x^2} + \frac{b}{(d-x)^2}$ is the function whose minimum we seek.

Ans. $x = \frac{da^{\frac{1}{3}}}{a^{\frac{1}{3}} + b^{\frac{1}{3}}}$.

Joseph William Mellor (1902/1922)

(14) A privateer has to pass between two lights, A and B, on opposite headlands. The intensity of each light is known and also the distance between them. At what point must the privateer cross the line joining the lights so as to be illuminated as little as possible. Given the intensity of the light at any point is equal to its illuminating power divided by the square of the distance of the point from the source of light. Let p_1 and p_2 respectively denote the illuminating power of each source of light. Let a denote the distance from A to B. Let x denote the distance from A to the point on AB where the intensity of illumination is least; hence the ship must be illuminated $p_1/x^2 + p_2/(a - x)^2$. This function will be a minimum when $(p_1/p_2)^{\frac{1}{3}} = x/(a - x)$; $\therefore x = ap_1^{\frac{1}{3}}(p_1^{\frac{1}{3}} + p_2^{\frac{1}{3}})^{-1}$.

Edward Olney (1871)

Ex. 19. A privateer wishes to get to sea unobserved, but has to pass between two lights, A and B, on opposite headlands, the distance between which is a. The intensity, at a unit's distance, of A is b, and of B, c. At what point must the privateer cross the line joining the lights, so as to be as little in the light as possible; it being understood that the intensity of a light at any point equals its intensity at a unit's distance divided by the square of the distance from the light.

SUG.—Letting $x =$ the distance from A, the function is $u = \frac{b}{x^2} + \frac{c}{(a - x)^2}$.

$$x = \frac{ab^{\frac{1}{3}}}{b^{\frac{1}{3}} + c^{\frac{1}{3}}}.$$

Edward Albert Bowser

21. A privateer wishes to get to sea unmolested, but has to pass between two lights, A and B, on opposite headlands, the distance between which is c. The intensity of the light A at a unit's distance is a, and the intensity of B at the same distance is b; at what point between the lights must the privateer pass so as to be as little in the light as possible, assuming the principle of optics that the intensity of a light at any distance equals its intensity at the distance one divided by the square of the distance from the light.

$$x = \frac{ca^{\frac{1}{3}}}{a^{\frac{1}{3}} + b^{\frac{1}{3}}}.$$

James Morford Taylor (1897)

31. A privateer must pass between two lights A and B on opposite headlands, the distance between which is c. The intensity of light A at a unit's distance is a, and the intensity of light B at a unit's distance is b. At what point must a privateer pass the line joining the lights so as to be as little in the light as possible, assuming the principle of optics, that the intensity of a light at any point is equal to its intensity at a unit's distance divided by the square of the distance of the point from the light?

Ans. $x = \dfrac{ca^{\frac{1}{3}}}{a^{\frac{1}{3}} + b^{\frac{1}{3}}}$.

William Shaffer Hall

31. A privateer must pass between two lights A and B on opposite headlands, the distance between which is c. The intensity of light A at a unit's distance is a, and the intensity of light B at a unit's distance is b. At what point must a privateer pass the line joining the lights so as to be as little in the light as possible, assuming the principle of optics, that the intensity of a light at any point is equal to its intensity at a unit's distance divided by the square of the distance of the point from the light?

Ans. $x = \dfrac{ca^{\frac{1}{3}}}{a^{\frac{1}{3}} + b^{\frac{1}{3}}}$.