How to Measure the Value of $\operatorname{Pi}(\pi)$

Some terminology in this unit:
ratio
I think of a ratio as an unperformed division, such as $a / b$ or a divided by $b$, where $a$ and $b$ are numbers (but b can't be zero!)
circumference of a circle
diameter of a circle
positive integer
(positive) rational number
irrational number
the distance around a circle's border, abbreviated C
the distance from edge to edge of a circle, going through the center; also, the longest chord in a circle; abbreviated d the counting numbers $1,2,3, \ldots$
a number of the form $a / b$ or a divided by $b$, where both $a$ and $b$ are positive integers
a number that cannot be written $a s a / b$ with $a$ and $b$ both integers (V2 and $\varphi$, the golden ratio, are examples)
$\mathrm{Pi}(\pi)$ is the ratio of the circumference of a circle to its diameter, namely, $\mathrm{C} / \mathrm{d}$.

Pi is an irrational number (not a mixed number, so it can't be written as $\mathrm{a} / \mathrm{b}$, where $a$ and $b$ are integers), so in all practical applications we use only its approximations.

When people were using mainly common fractions, $3^{1 / 7}$ was the most popular approximation $(22 / 7)$.

Now, 3.14 is probably the most common. In many everyday applications, even plain old 3 is good enough.

But how to get any of these?

Does one really need mathematics, or can one simply measure pi?

Let's see if we can measure it!

Supplies needed (pictures on next slides!)

- poster board or piece of thin cardboard (for example, from a cereal box)
- ruler
- compass
- round toothpick or paper clip
- scissors
- pencil
- one sheet of paper cut into a few strips and taped together to make one long strip
- scotch tape
- calculator


1. You need a piece of cardboard.

2. Set your compass at any radius you want, and draw a circle on the cardboard. Be sure that the center of the circle is clearly marked.
3. Draw a radius on your circle, and very carefully measure its length in centimeters. Write the length on the radius.

4. You need to make a straight track to roll your circle on. You can do this by cutting a piece of paper into strips, drawing a straight line down each strip, and, as you will see, matching the lines, and taping the strips together to make a long roadway.

5. Tape your long strip of paper on a flat table. Near one end of the line, make a tick mark and write "start" on the paper. Carefully put the circle on the line. The radius of the circle that you drew should be vertical and should touch the line. Now roll the circle a full 360 degree turn along the line, until the radius is again in vertical position (see next slide).

6. Stop your circle when the radius is again in vertical position. Make a tick mark and label it "end".

7. The length of the line from the "start" to the "end" is the circumference of the circle. Do you see it? Now comes the awesome part! Carefully measure the length.
8. Use a calculator. Divide the circumference by twice the radius. This is your estimate of the value of pi! We recommend you try at least two trials.

An example

I need a very precisely cut-out circle, and I measure its radius very carefully.
Radius $=5.6 \mathrm{~cm}$
2 times radius = diameter $=11.2 \mathrm{~cm}$

First trial. Circumference $=34.9 \mathrm{~cm}$
(using a calculator) $\mathrm{C} / \mathrm{d}=34.9 / 11.2=3.1160714$ (Notice that when we divide 34.9 by 11.2, cm cancels, so we get a number without units attached.)

Second trial. (Roll your circle once more along the straight line, as before, making a new tick mark at the end.)
Circumference: 35.0 cm
$35.0 / 11.2=3.125$

Now you and those in your breakout room can make a table of your findings:

| Name | circumference C | diameter $d$ | c/d |
| :--- | :--- | :--- | :--- |
| PB | 34.9 cm | 11.2 cm | 3.1160714 |
|  | 35.0 cm | 11.2 cm | 3.125 |
|  | $\ldots$ | $\ldots$ | $\ldots$ |

In your group, you may compute the average of C/d. Is it close to 3.1415926 ?

Of course, since $\pi=C / d$, we can solve for $C$, and we get the standard formula $C=\pi d$. So, if you know the diameter of a circle (or its radius), you can find its circumference.

Another way to look at pi: If you wrapped a circle's diameter around its circumference, you would need a little more than 3 diameters to make it all the way around, actually about 3.14 diameters!

## More about pi

1. Just as all equilateral triangles are similar (the same shape, but not necessarily the same size), so are all circles on a plane similar. This means that the ratio of the circumference of a circle to the diameter is always the same, independent of the size of the circle.
2. What exactly is this ratio, which has been labeled pi (the Greek letter $\pi$ )? For centuries, people tried to find out. Practically, you may use different approximations, $3,3^{1} / 7,3.14, \ldots$, and so on. But what is its true value?
3. We know now that $\pi$ is irrational, just as square root of 2 is irrational. This means that $\pi$ cannot be written exactly as a mixed number. But there are efficient methods of getting as many digits as you want. As a challenge, people using computers have calculated millions and millions of digits.
4. A few digits of pi:
3.141592653589793238462643383279502 88419716939937510582097494459230781 64062862089986280348253421170679821 48086513282306647093844609550582231 72535940812848111745028410270193852 11055596446229489549303819644288109 75665933446128475648233786783165271 20190914564856692346034861045432664 82133936072602491412737245870066063 15588174881520920962829254091715364 36789259036001133053054882046652138 41469519415116094330572703657595919 53092186117381932611793105118548074 46237996274956735188575272489122793

Let's look at some interesting facts about pi: https://www.piday.org/pi-facts/

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And here is the pi search page. You can put in any string of digits, and it will see if that string can be found in pi.
https://www.angio.net/pi/

Let's look at some interesting facts about pi: https://www.piday.org/pi-facts/
pi in the Bible
1 Kings 7:23 King James
And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it round about

And here is the pi search page. You can put in any string of digits, and it will see if that string can be found in pi.
https://www.angio.net/pi/piquery

And of course we celebrate Pi Day, March 14!

## Bye bye $\pi$

