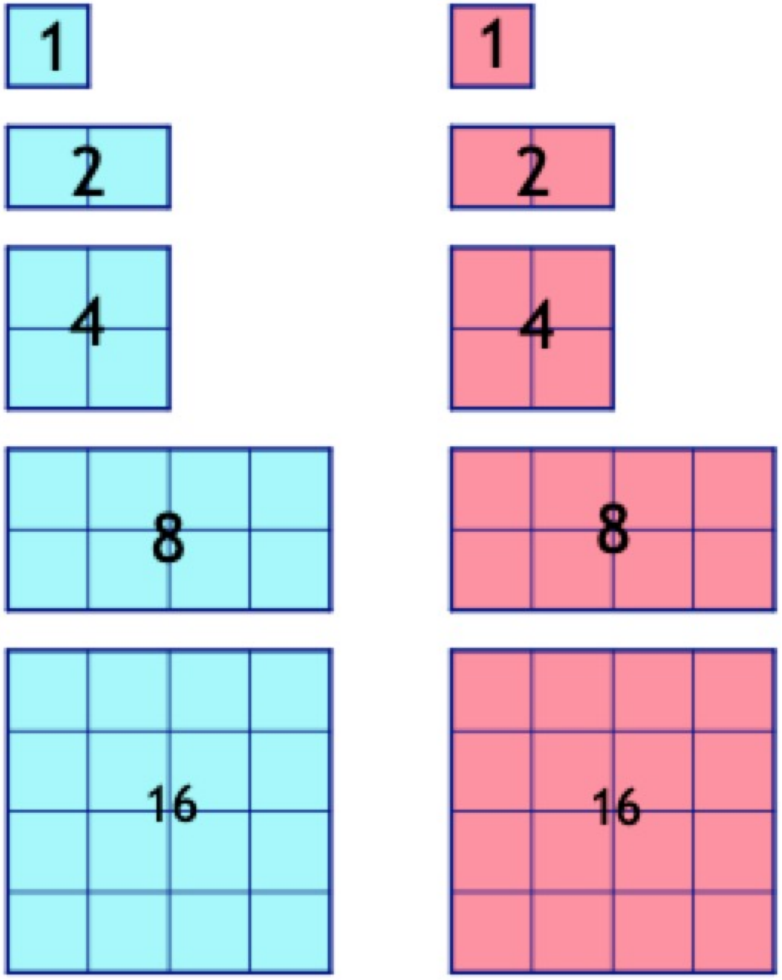


Arithmetic with Manipulatives

THE LESSON

Part 1

This lesson has been taught in kindergarten and first grade. In kindergarten, only manipulatives [1], [2], [4], and [8] were used, and they were cut out beforehand by the teacher. She also gave each child a black “mat” (piece of construction paper) on which to make shapes and numbers. Children then knew what to include when they were counting, namely, the manipulatives (square inches) that were on the mat.

INTRODUCTION

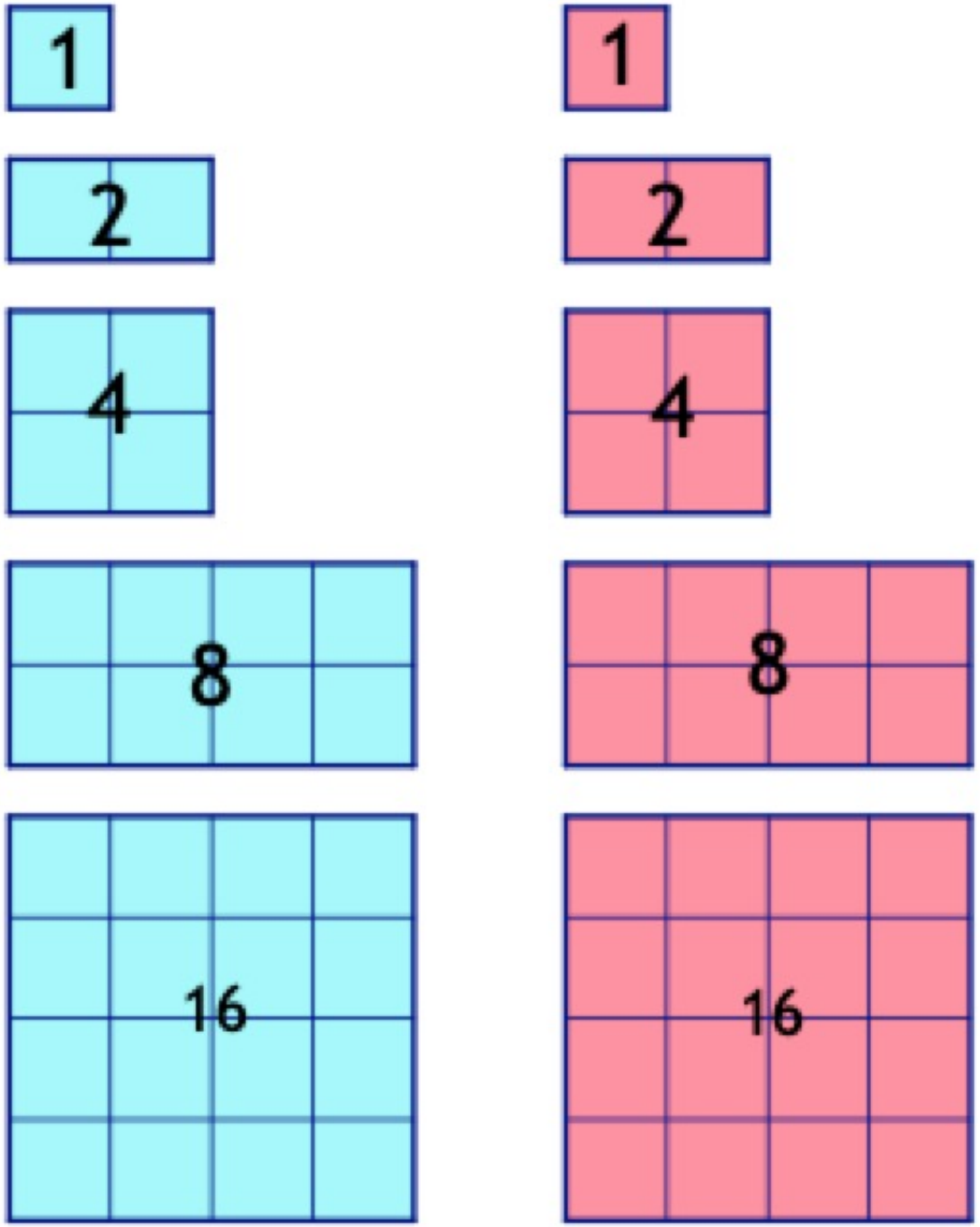
In the first grade, a large amount of time is spent on small whole numbers (less than 30), when children learn basic addition and subtraction facts. This teaching is done mainly with manipulatives such as base ten blocks, Unifix cubes, counters, Cuisinaire rods, and so on.

Here we suggest a different set of manipulatives, which can possibly replace the other ones.

2. MANIPULATIVES

One set of manipulatives consists of five rectangles (three of them are squares) labeled with the numbers 1, 2, 4, 8, and 16. The label describes the area of the rectangle measured in square inches. So the set looks like the ones shown in illustration 1.

The pieces may be of different colors, and they may be decorated. But the numbers written on them should be legible for the children. Every counting number up to 31 can be uniquely represented as an area (measured in square inches) built from these rectangles. Rectangles are shown below as [1], [2], [4], [8], and [16].


1
2
4
8
16
1
2
4
8
16

1 [1]
2 [2]
3 [2] [1]
4 [4]
5 [4] [1]
6 [4] [2]
7 [4] [2] [1]
8 [8]
9 [8] [1]
10 [8] [2]
11 [8] [2] [1]
12 [8] [4]
13 [8] [4] [1]
14 [8] [4] [2]
15 [8] [4] [2] [1]
16 [16]
17 [16] [1]
18 [16] [2]
19 [16] [2] [1]
20 [16] [4]
21 [16] [4] [1]
22 [16] [4] [2]
23 [16] [4] [2] [1]
24 [16] [8]
25 [16] [8] [1]
26 [16] [8] [2]
27 [16] [8] [2] [1]
28 [16] [8] [4]
29 [16] [8] [4] [1]
30 [16] [8] [4] [2]
31 [16] [8] [4] [2] [1]

We recommend that for first grade, each child has two sets (10 pieces) , in two different colors, so that up to two numbers up to 31, or one number up to 62, can be represented.

3. CREATING A SET

We suggest that in first grade each child creates his or her own set.
The amount of work done by a child should be determined by the teacher.
Because reasonable precision is needed, we suggest that the rectangles be precut from stiff colored poster board or card stock.
Each child gets a large envelope on which he or she writes his or her name, together with some rectangles (only a few at a time).
.

The size of the area of each rectangle is discussed (the smallest is a square inch), and each child writes his or her name on one side of the rectangle and the appropriate number (possibly with help from the teacher).
The side with the name may be decorated.

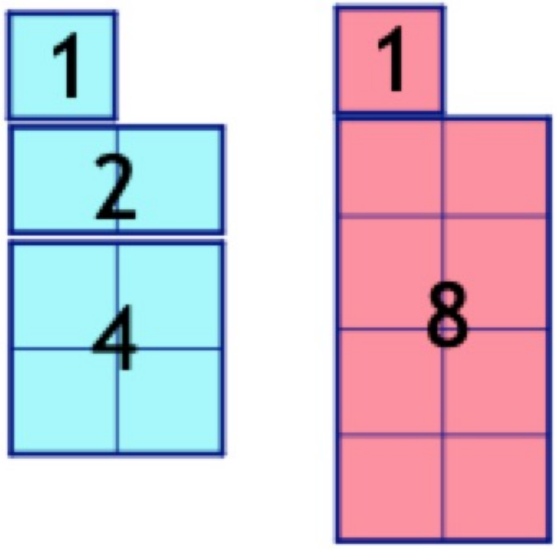
This activity should be carried out (possibly for 3 or 4 times) during one week until each child has two sets of manipulatives in the envelope

4. ACTIVITIES

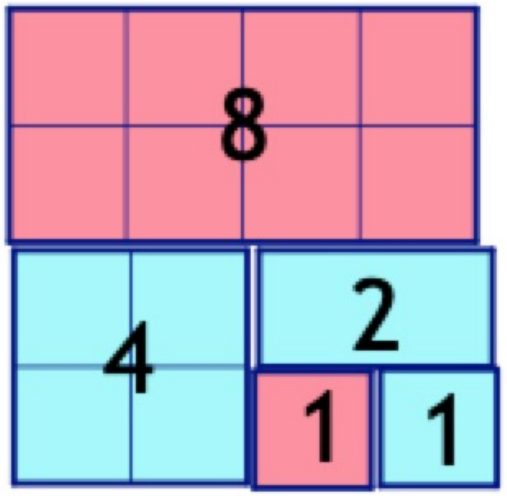
Any arithmetic activity using manipulatives may be carried out with this set.

We suggest:

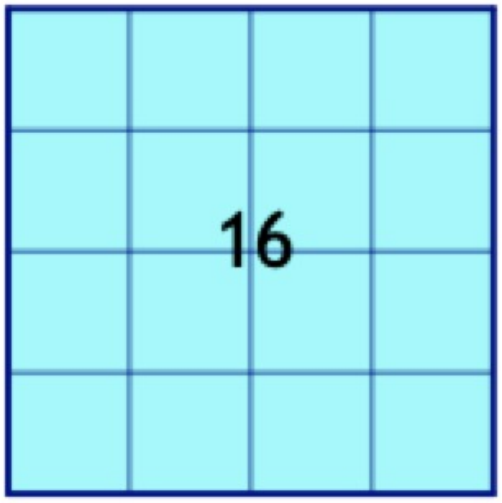
- making numbers (e.g., the teacher says, “Make 7. Make 9. Add them. Read the number.” (see next slide)



A. Make 7. Make 9.



B. Add them.



C. Regroup.

MORE ACTIVITIES

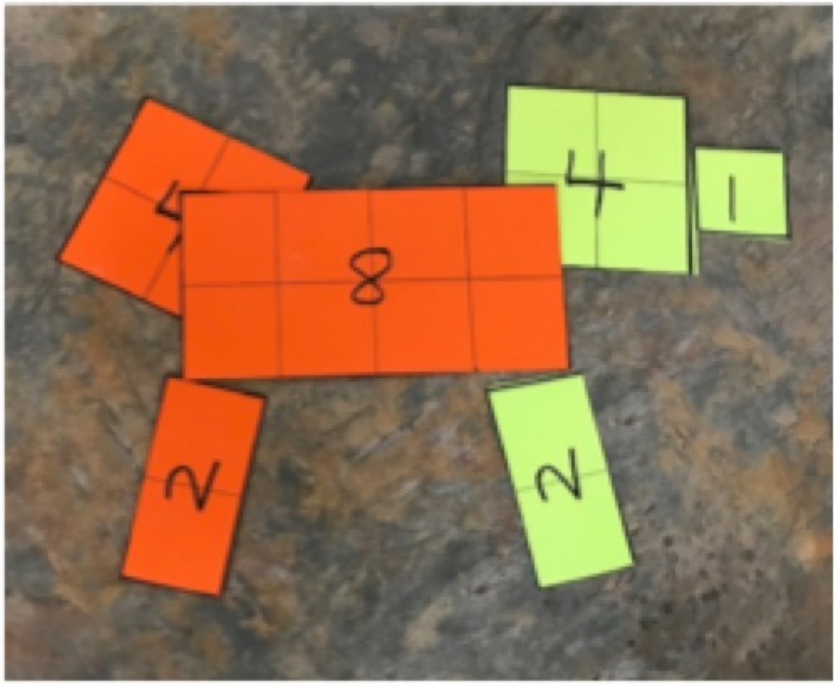
- counting the total given. (When [4] and [2] are present, the total is 6, not 4 + 2! If needed at first, calculators may be used.)
- asking if the representation of a number is unique. E.g., are there two ways to make 8?
- asking, by using two colors, how many ways can I make a number?

- asking what areas I can make: 3 x 3 square inches? (yes, but how?)
  5 x 5? hmmm…
  6 x 6? yes!

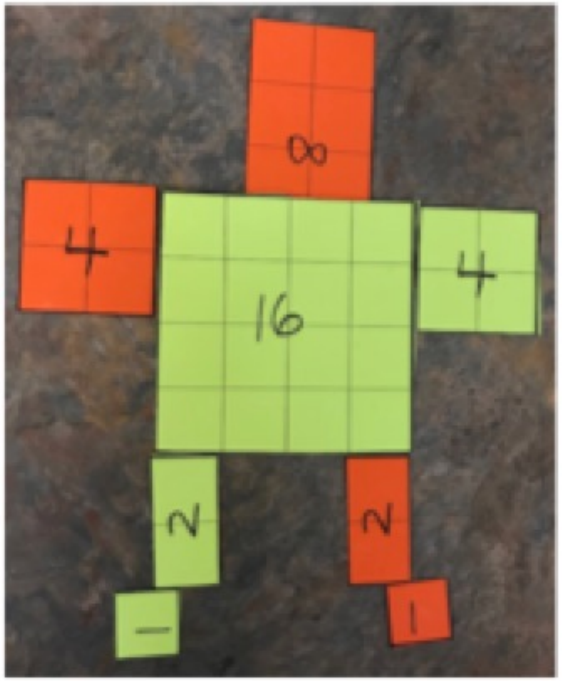
5 x 5?

6 x 6

- addition (with regrouping)
- subtraction with regrouping
- learning basic addition and subtraction facts
- using the sets to help with mental calculations
- making pictures (a dog, a cat, a human, etc.)



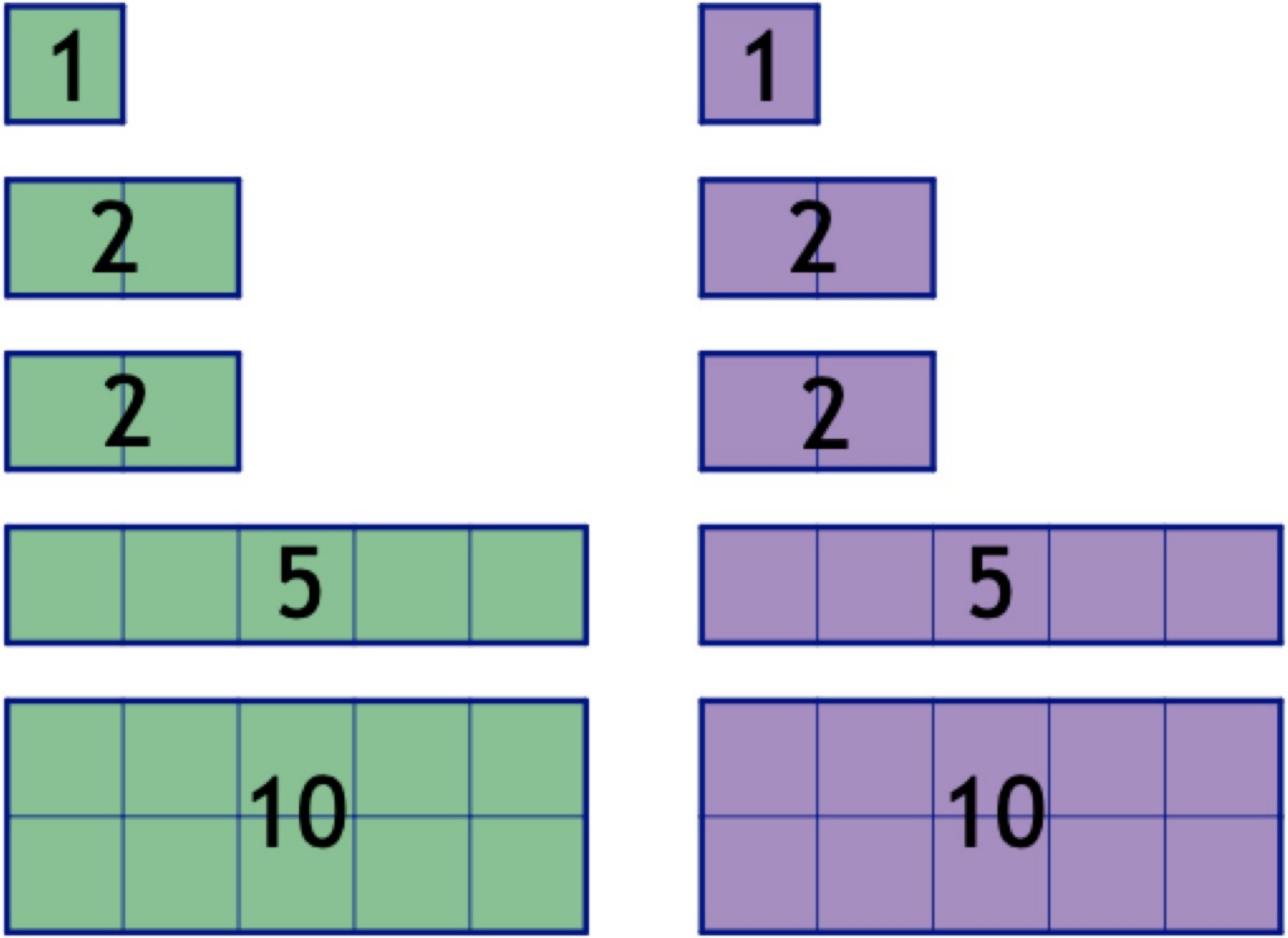
animal?



human?

Part II: Another set of manipulatives

The illustration on the next slide shows an example of manipulatives which can be used in teaching the concept of regrouping in algorithms for addition and subtraction in base 10. They are rectangles built from squares one inch by one inch, just as in the materials in Part 1. Each child should have one set of 10 rectangles (see illustration). We suggest, as we did in Part 1, that children help in making these manipulatives, decorate them, and wite the number of square inches on each one (1, 2, 5, 10).


1
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1
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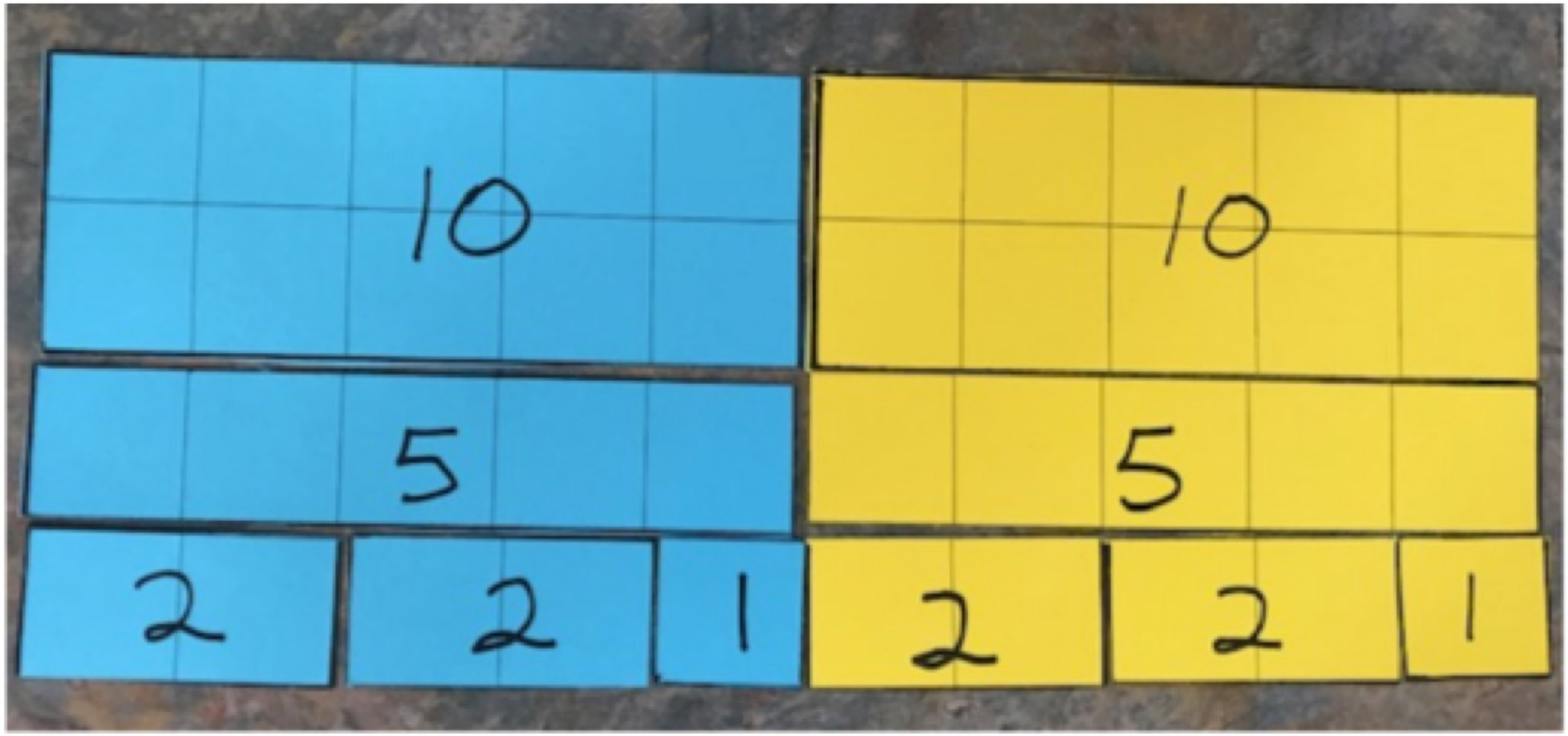
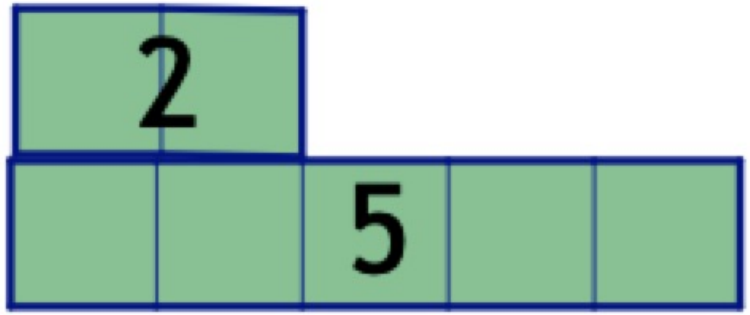
2
5
10



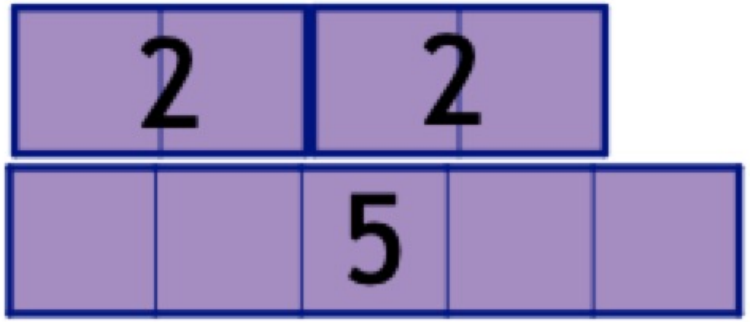
the set of pieces I made

MAIN ACTIVITIES

(1) Adding two numbers smaller than 10, with regrouping.
Example: 7 + 9
(a) How to make two numbers?



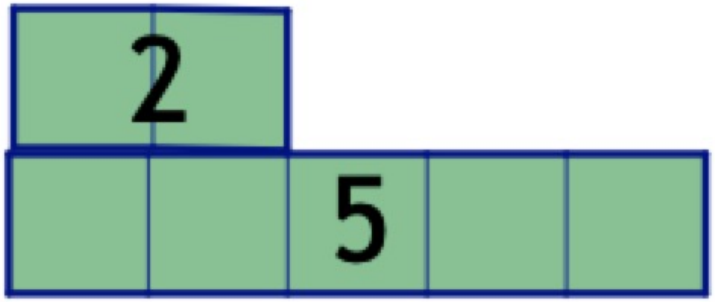
Making 7



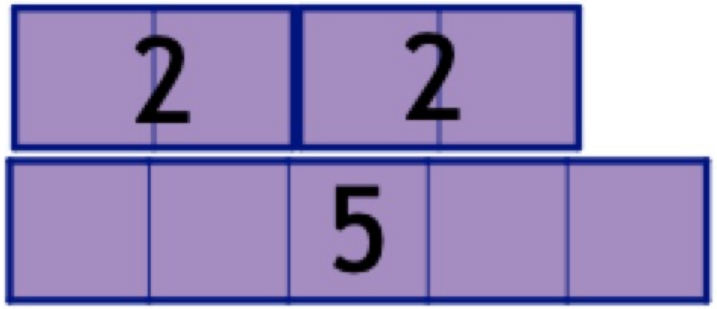
Making 9

MAIN ACTIVITIES

(1) Adding two numbers smaller than 10, with regrouping.
Example: 7 + 9
(a) How to make two numbers?

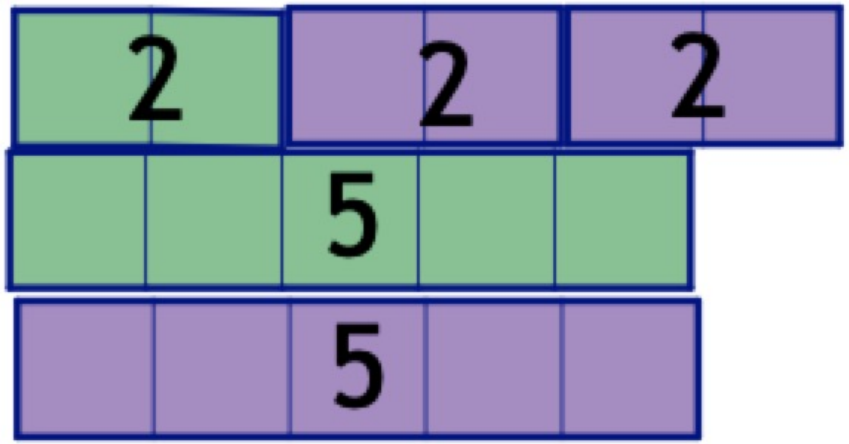


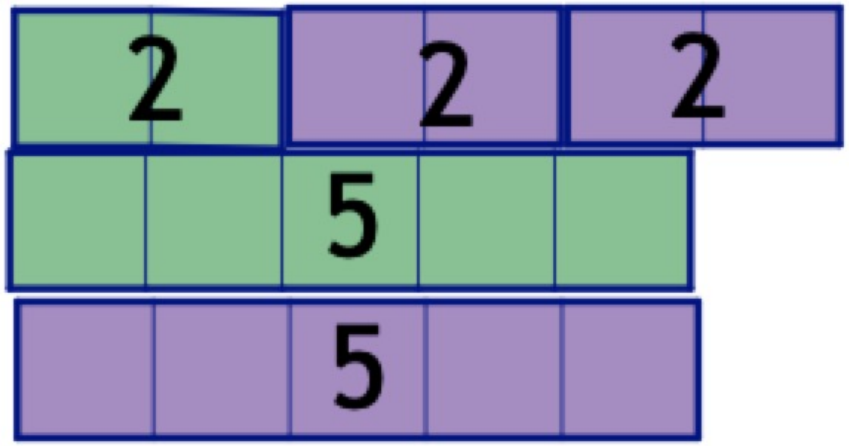
Making 7



Making 9

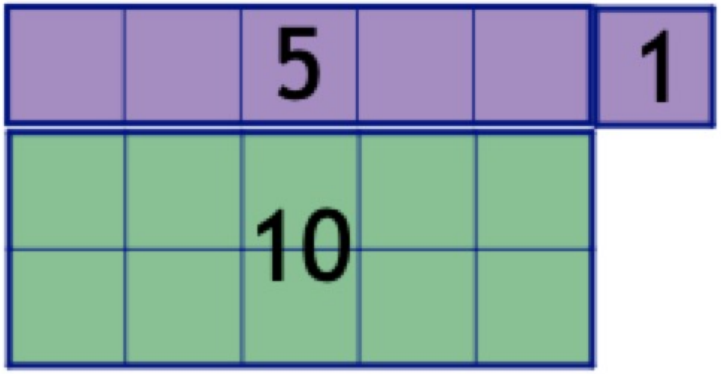
(b) How to add them?

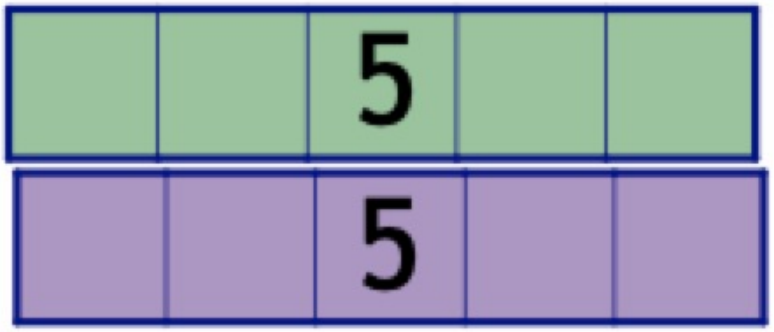




c. How to regroup?

Two ways:





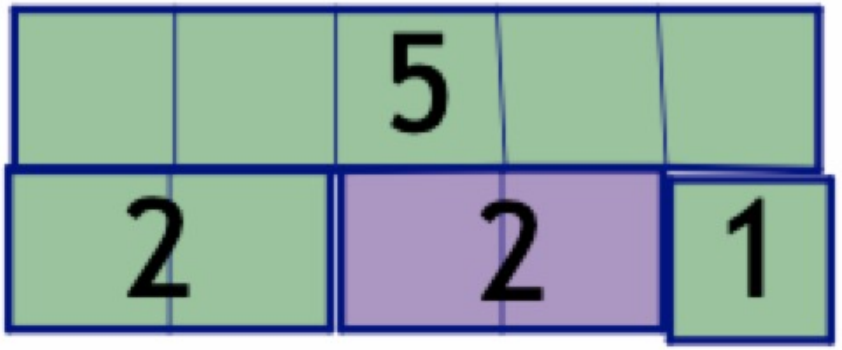
(2) Subtracting a one digit number from 10.

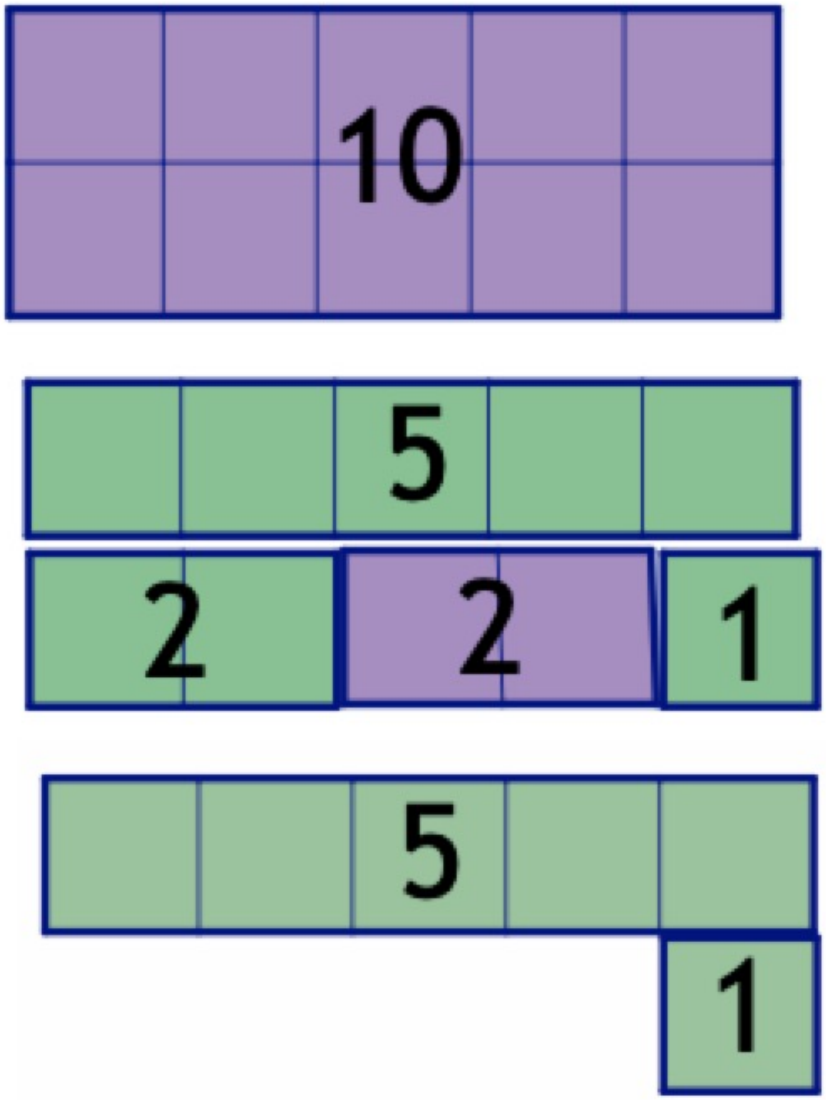
Example:

(a) Regrouping 10

10 = 5 + 5

= 5 + 2 + 2 + 1





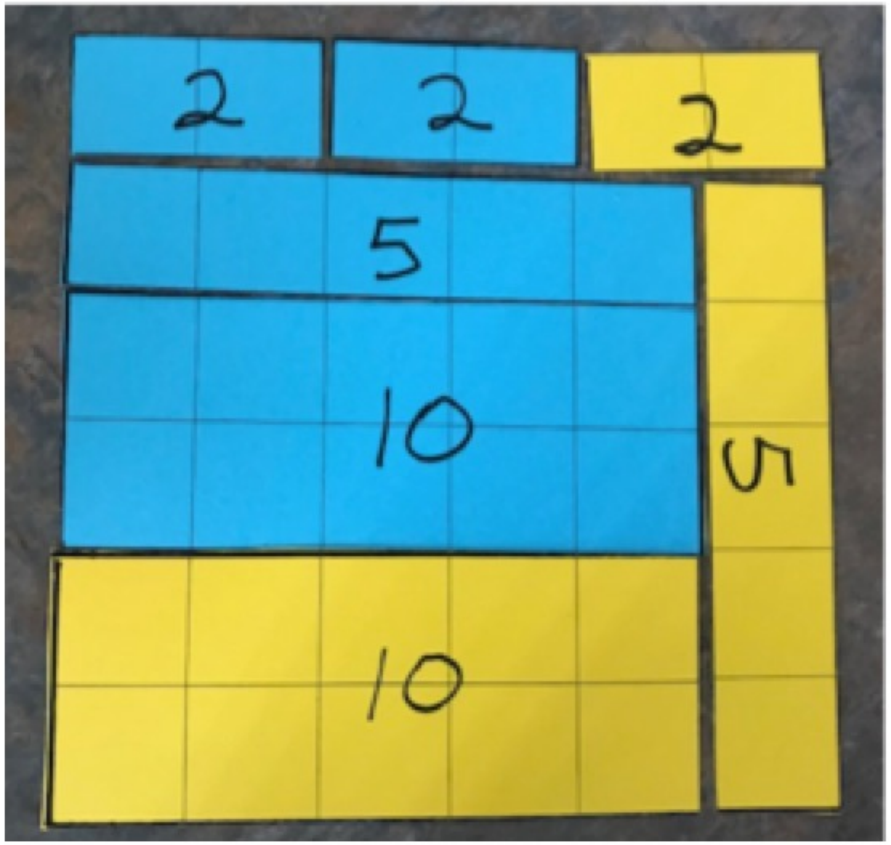
(b) Canceling:

$10 - 4$

$= (5 + 2 + 2 + 1) - (2 + 2)$

$= 5 + 1$

4. What squares and rectangles can you make?

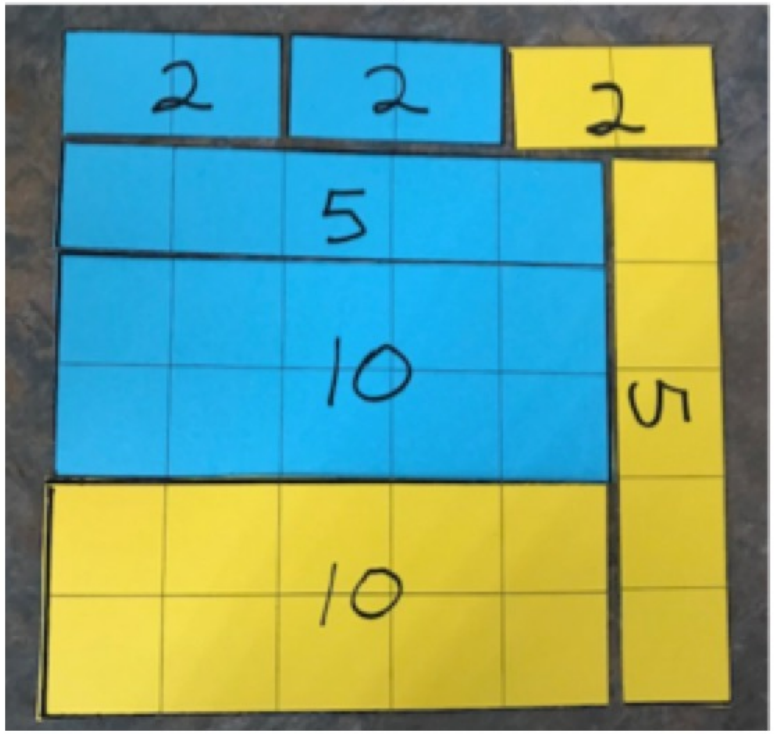


One example: A 6 by 6 square

COMMENTS

- An explanation of “why” should be done in terms of areas. “We replace one figure by another of the same area.”

- The statement that base 10 requires only that we “replace one ten by ten ones” is incorrect and misleading. “You may replace 10 by the sum of any numbers that add up to ten.”



Two six by six squares!

Now it's your turn! Some tasks for you to try!

1. First, make both sets (1, 2, 4, 8,16 aka "base 2", and 1, 2, 5, 10 aka "base 10")
2. With the first set, make all the numbers from one through 31 (or 62). Can you make any number in more than one way? (I will show you how a kindergartner actually did!)
3. Again, with the first set, show how to solve 12 – 8. Can you regroup?
4. With the first set, add 6 + 7.
5. With the second set, can you show how to count, beginning with one? How high can you count? In how many ways can you make each number?
6. With the second set, what is 11 - 6?
7. What about 1 + 2 + 3 + 4?
8. Now make up your own problems!


THE END