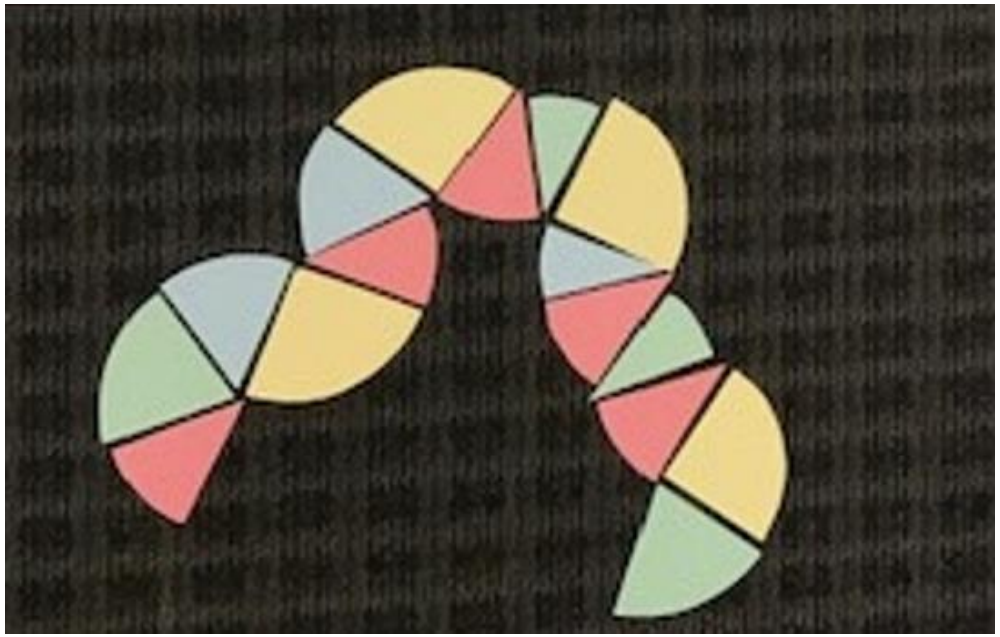
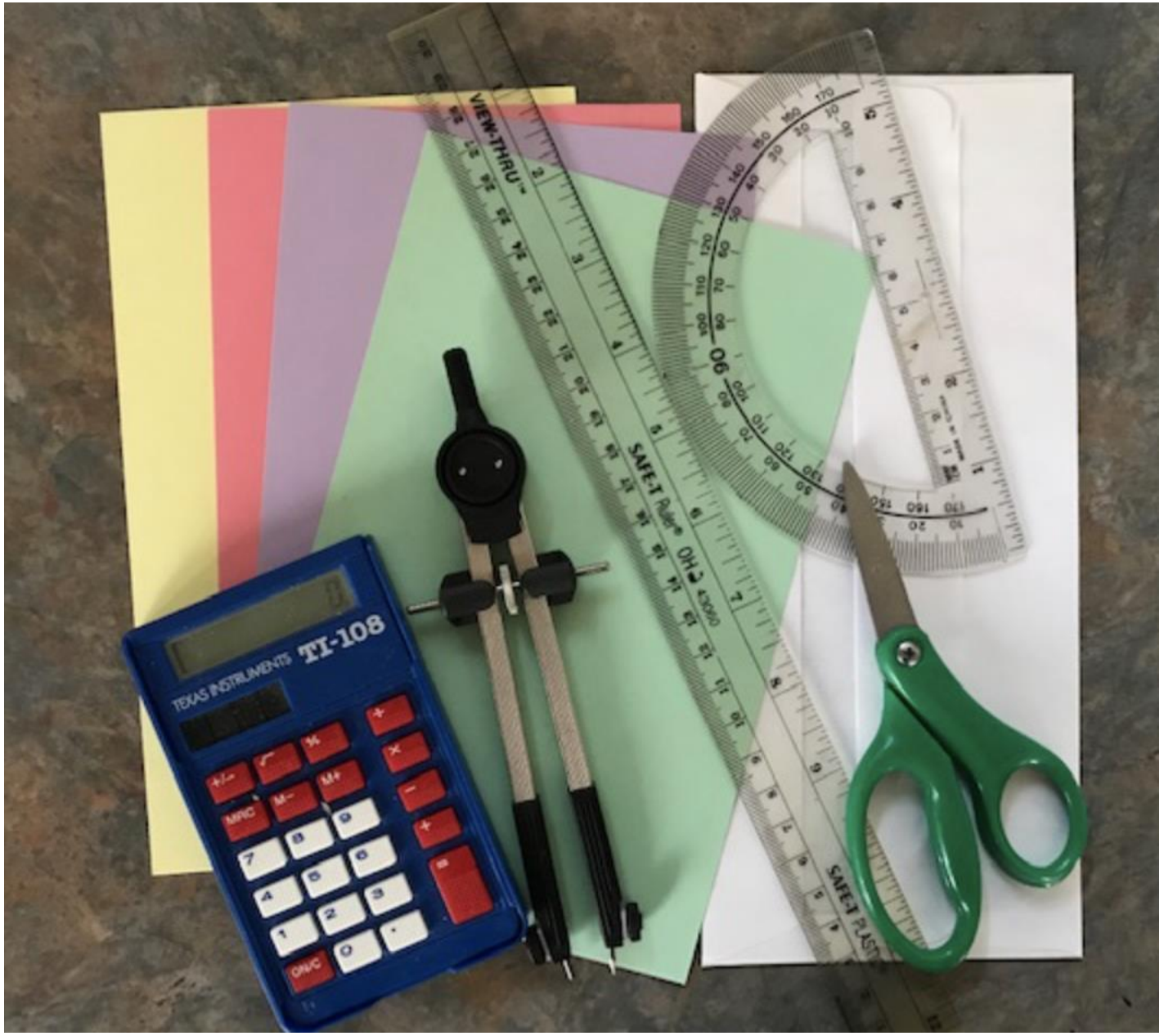


Fractions of a Circle
AKA
Caterpillar

Materials for this unit

- Four brightly colored 5 inch by 8 inch index cards, four different colors (you will actually need four 5 inch by 5 inch squares)
- Envelope to keep pieces in
- Compass
- Ruler
- Protractor
- Scissors
- Calculator



The task

Children work in pairs (in face-to-face instruction) or alone (online). Each pair (or each person) gets four 5-inch by 8-inch index cards, or 4 squares, 5 inches by 5 inches, of different colors (for example, yellow, pink, lavender, and green), one or two envelopes, and a set of tools. The children are asked to draw and cut out four circles, each five inches in diameter. The circles need to be divided into sectors* as follows:

- Yellow—four quarters (¼ of a circle each)
- Pink—eight equal parts ($^1/_8$ of a circle each)
- Lavender—eight unequal parts (four parts, 1/6 of a circle each; four parts, 1/12 of a circle each)
- Green—eight unequal parts (two parts 1/5 of a circle each; six parts 1/10 of a circle each)

Each person gets half (or all) of all the pieces and keeps them in his or her envelope.

*A sector of a circle is a pie-shaped part of a circle.

Method

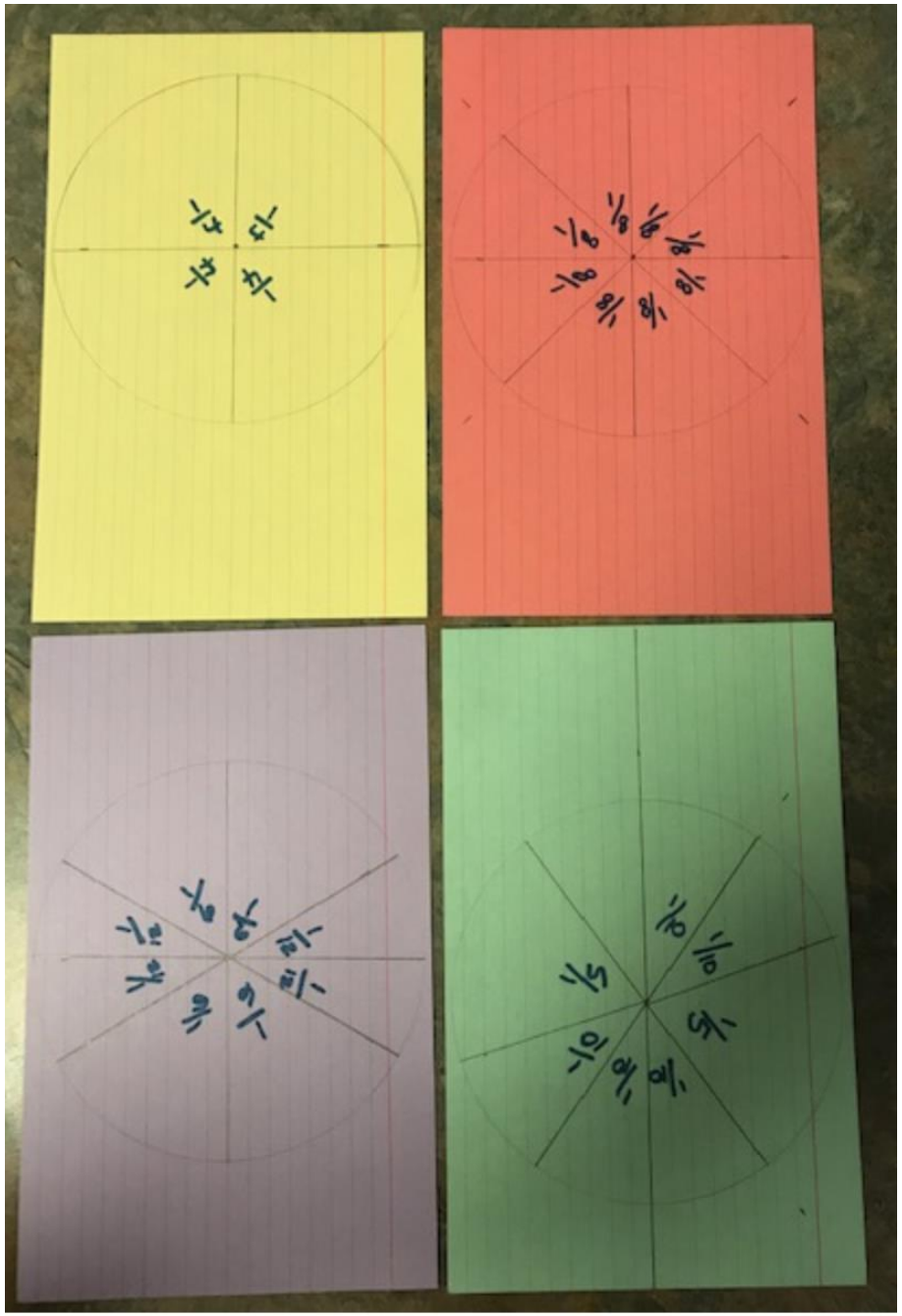
Compute the appropriate angles (shown in the table below). Find the center of the lined side of each card, and set your radius to 2½ inches. Draw the four circles. Using a protractor, draw the angles within the circles. Carefully cut out the circles and cut them into sectors. (Pictures on next slides)

Part of a circle	¼	1/8	1/6	1/12	1/5	1/10
Angle in degrees	90°	45°	60°	30°	72°	36°

(If children have not practiced with the tools, they may start by drwing the plans on normal whitel paper before copying them onto the index cards.)

My four circles before I cut them

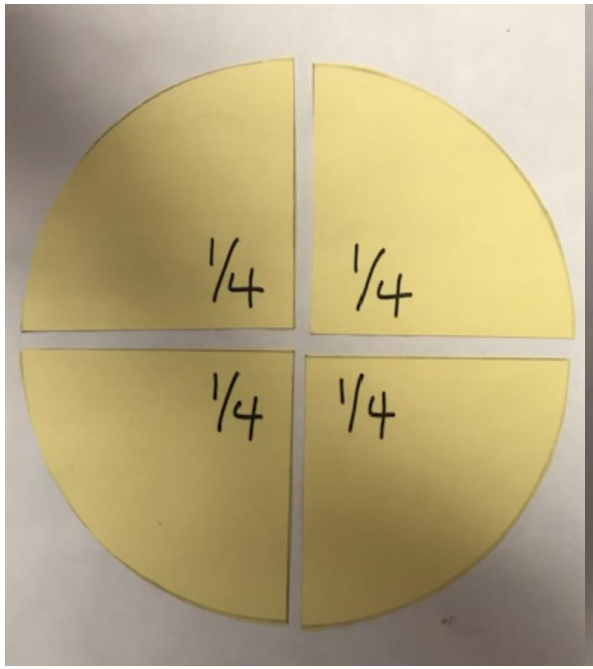
Yellow: Four quarters, ¼ of a circle each

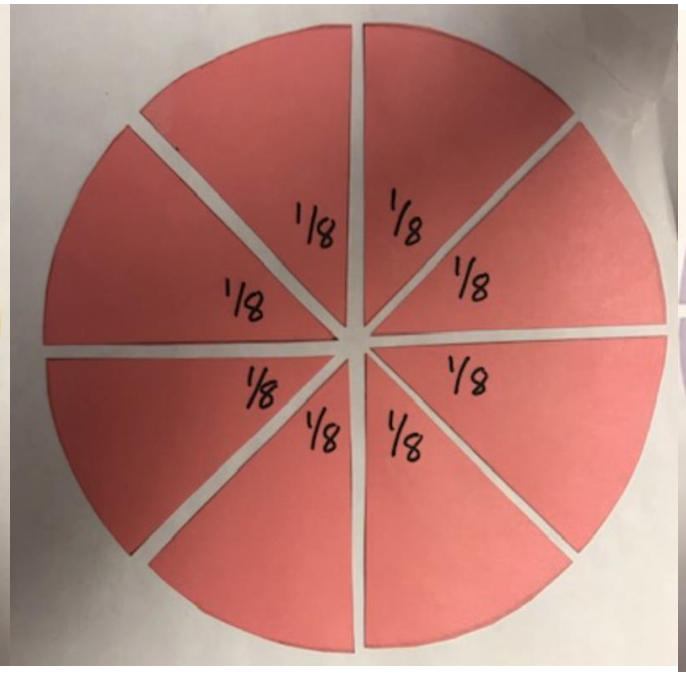


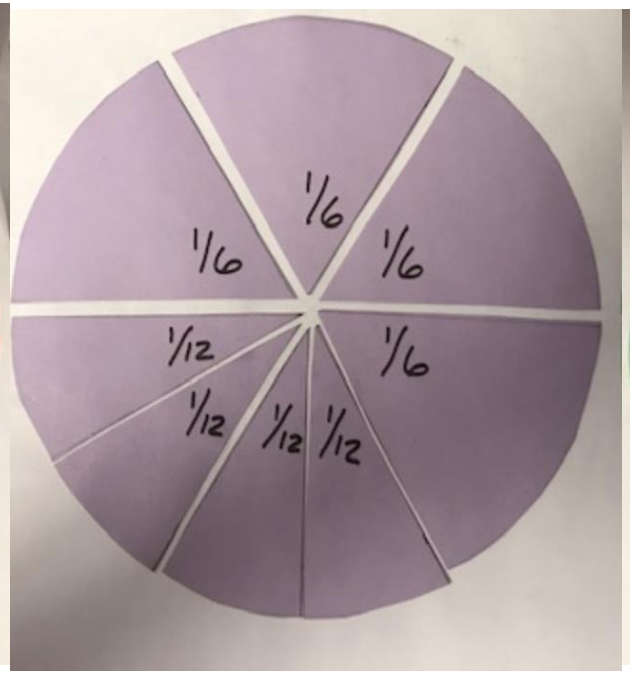
Pink—eight equal parts ($^1/_8$ of a circle each)

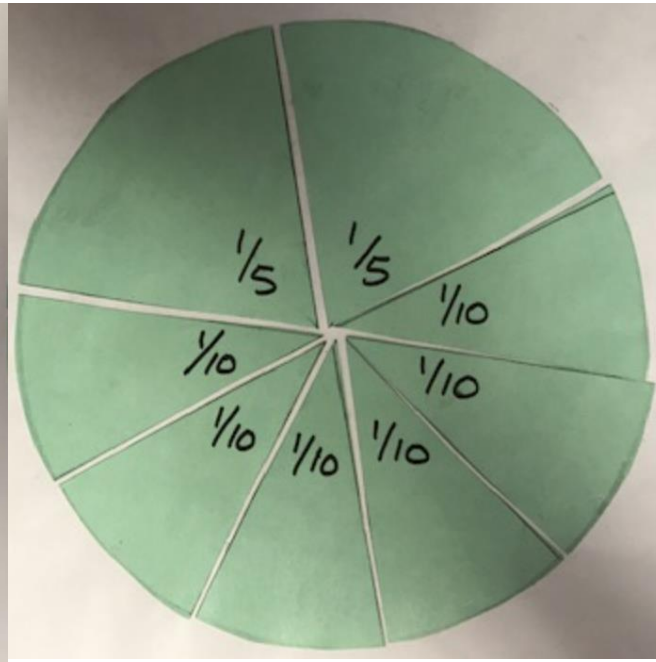
Lavender—eight unequal parts (four parts, 1/6 of a circle each; four parts, 1/12 of a circle each)

Green—eight unequal parts (two parts 1/5 of a circle each; six parts 1/10 of a circle each)









My four cut-out circles with sectors labeled with fractions

- Yellow—four quarters (¼ of a circle each)
- Pink—eight equal parts (1/8 of a circle each)
- Lavender—eight unequal parts (four parts, 1/6 of a circle each; four parts, 1/12 of a circle each)
- Green—eight unequal parts (two parts 1/5 of a circle each; six parts 1/10 of a circle each)

Some activities you can do with the sectors

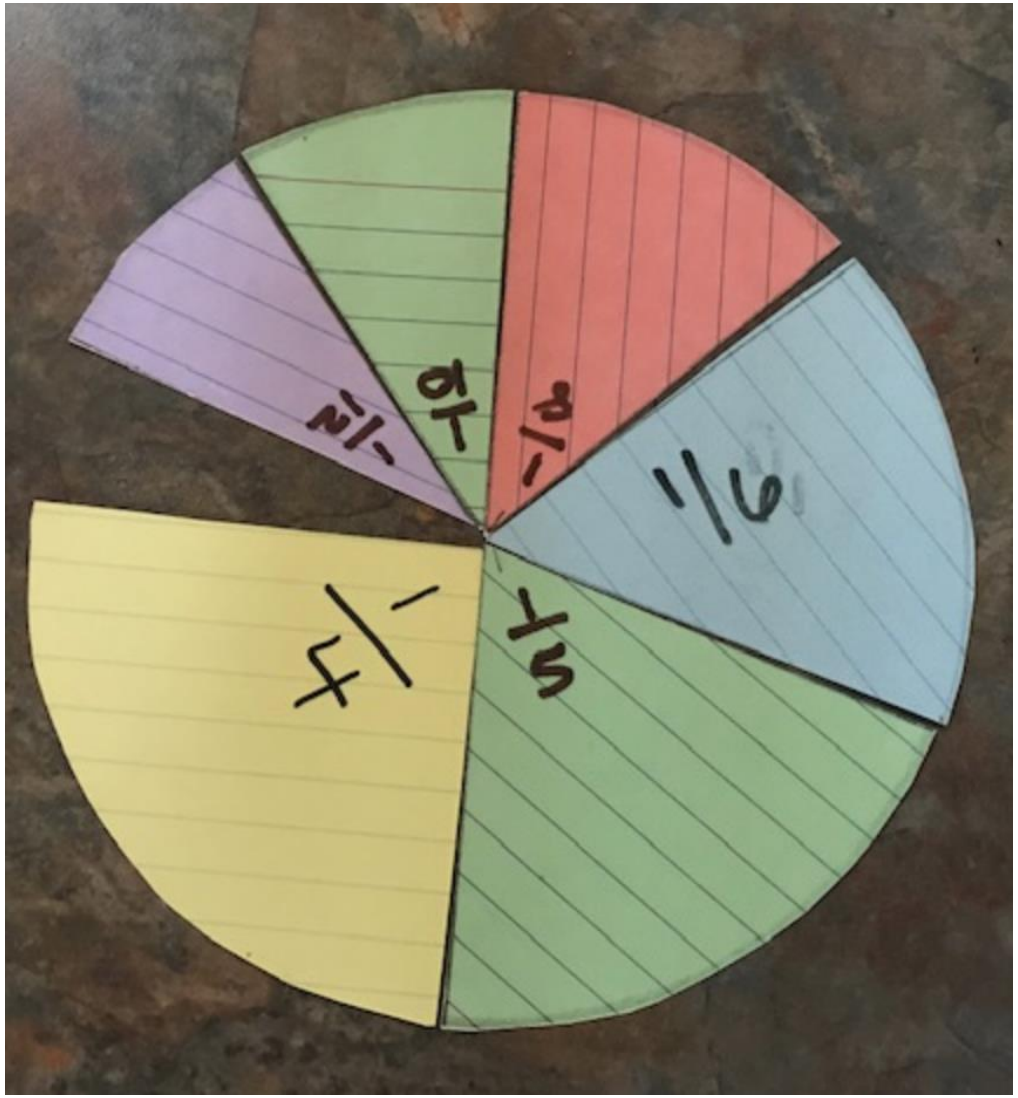
1. Order the sectors according to the sizes of the angles.

When we take one of each size, what is the total fractional measure?

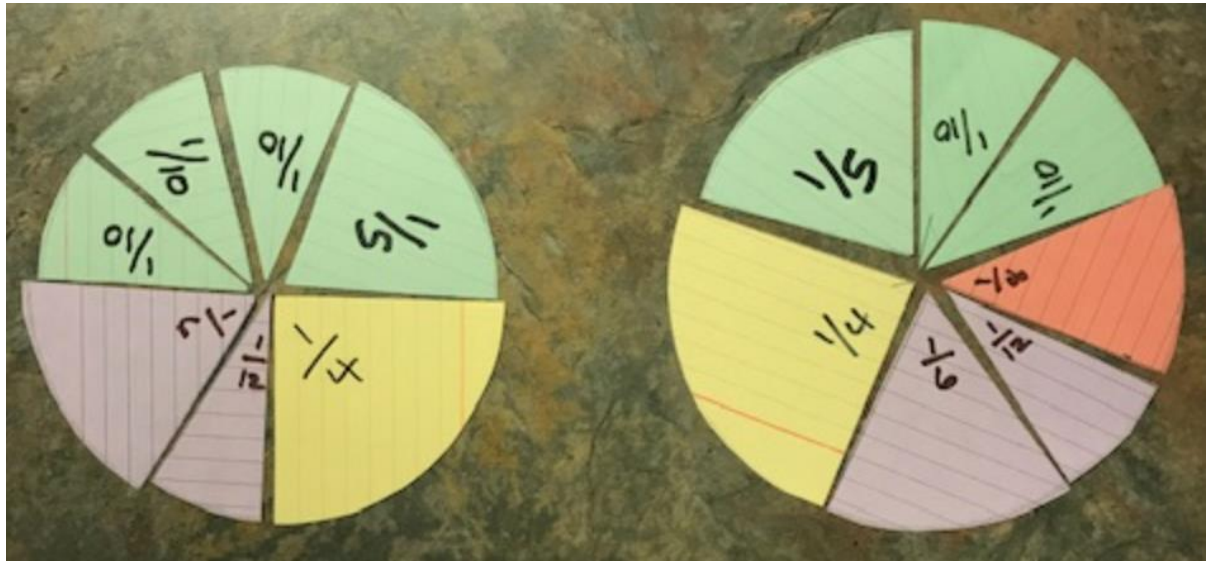
Yikes, a common denominator of 120!

¼ + 1/5 + 1/6 + 1/8 + 1/10 + 1/12 =
(30 +24 +20 + 15 + 12 + 10)/120 = 111/120
(divide numerator and denominator by 3) = 37/40

What is the total angle measure?
90° + 72° + 60° + 45° + 36° + 30° = 333°
and 333°/360° = 37/40! YAY MATH!



2. Make two whole circles in many different ways. Which sectors can you use? Two examples:



Part of a circle	¼	1/8	1/6	1/12	1/5	1/10
Angle in degrees	90°	45°	60°	30°	72°	36°

Let's see if each of these "circles" is really a circle, that is, if its pieces sum to 360 degrees.

left circle
1/6 +1/12 +1/4 + 1/5 + 1/10 + 1/10 + 1/10 =
60° + 30° + 72° + 90° + 36° + 36° + 36° = 360° yes!

right circle
¼ + 1/6 + 1/12 +1/8 +1/10 + 1/10 + 1/5 =
90°+ 60° + 30° + 45°+ 36° + 36° + 72°= 369° too big!

(If we wanted, we could have added the fractions instead!)

Extra credit: Can you get closer to 360° (but still different from 360°) with your pieces than I did with 369°? How?

3. Construct new angles when you are given their measure (examples: 186° = 90° + 60° + 36° (on the left); 60° + 45° + 36° = 141° (on the right).
But 136° cannot be made with the sectors. Can you prove it?

I can make 186°
186° = 90° + 60° + 36°

I can make 141°
141° = 60° + 45° + 36°

Extra credit problem. Can you make 136° with your pieces? Why or why not? Can you prove it?

4. Make interesting shapes such as a caterpillar.

a goofy animal

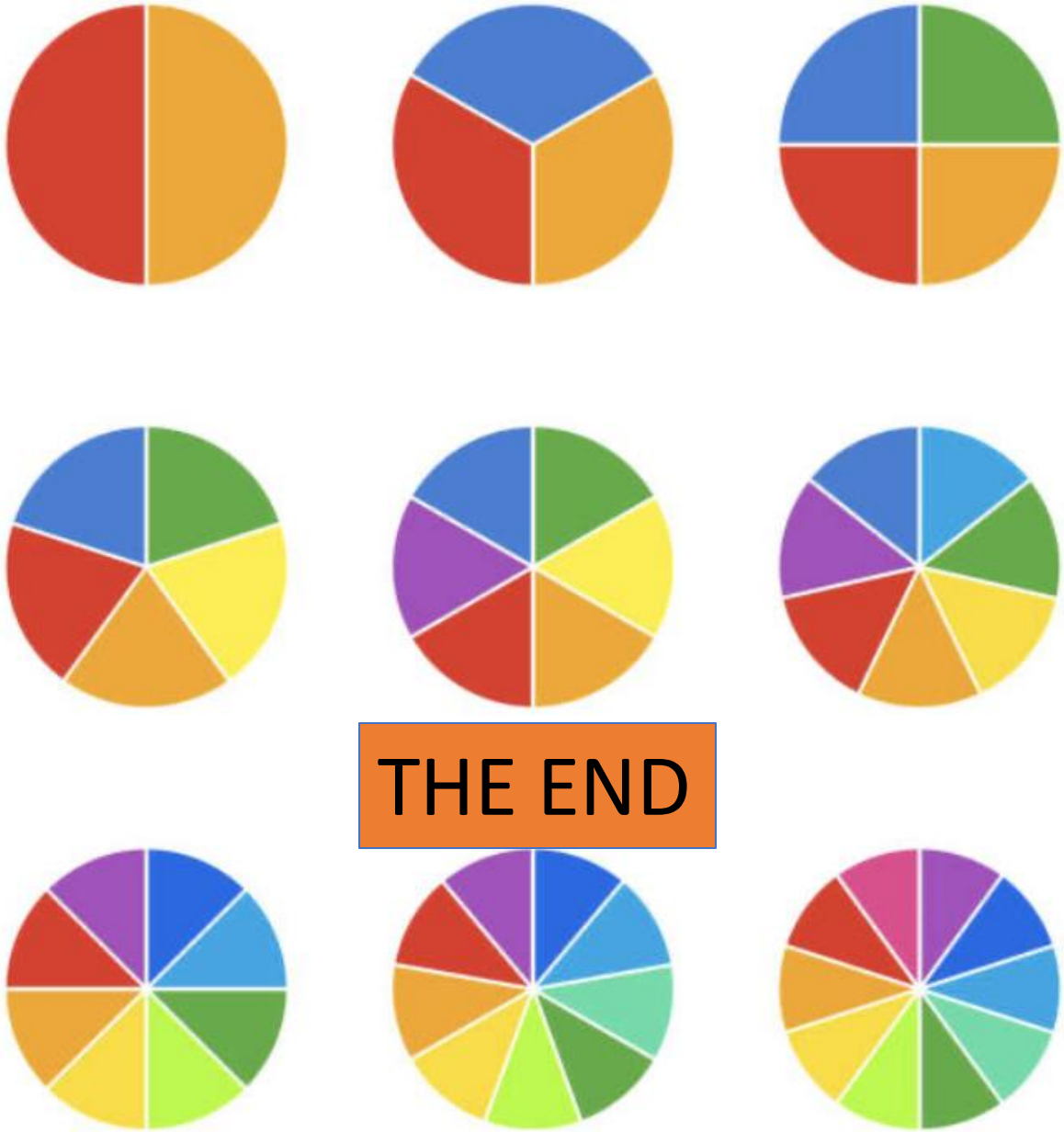
Some theoretical comments

This lesson deals mostly with skills using tools, but some comments about fractions can be relevant. In schools the word fraction is used in three different meanings:

(1) A fraction is part of an object. "Shade 1/3 of this circle." "Fractions of a circle."
(2) A fraction is a mathematical expression. "Write 0.2 as a common fraction." (1/5)
(3) A fraction is a number. "Add ½ and ¼." (You add numbers. The result is .75 or ¾.)

Usually the meaning is clear from the context, but children can get confused and not know if we are talking about numbers or objects.

We compute an angle (a number) of ¼ of a circle (an object) by multiplying 360° (a number) by ¼ (a number) or by dividing 360° by 4 (both numbers). Notice that the word angle is also used in three different meanings: a number, two rays, or the part of a plane between two rays.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